

八十八學年度轉學生入學考試

科目 微積分(經濟系) 共 3 頁第 1 頁 \*請在試卷【答案卷】內作答

科號: 123

I. 填充題 (共有九個空格, 每一空格八分, 請將答案依

甲、乙、丙 ... 次序寫出, 不需演算過程)

1. Let  $y = \sqrt[3]{(x+1)\sqrt{(x^2+1)}\sqrt[3]{(x^3+1)}}$ . Then  $\frac{dy}{dx}|_{x=0}$   
 = 甲.

2. Evaluate the following:

(a)  $\int_1^2 \frac{2x+1}{x(x+1)^2} dx =$  乙.

(b)  $\lim_{n \rightarrow \infty} \frac{1}{n} [2^{1/n} + 2^{2/n} + \dots + 2^{n/n}] =$  丙.

(c)  $\sum_{n=0}^{\infty} \left(1 - \frac{1}{n!}\right) \frac{1}{3^n} =$  丁.

3. The minimum value of  $\sqrt{(\cos x - \frac{1}{2})^2 + (\sin x - \frac{1}{2})^2}$   
 with  $x \in \mathbb{R}$  is 戊.

4. Let  $L$  be the tangent line to the curve  $x^3 + y^3 + 3xy^2 = 1$   
 at the point  $(0,1)$ . Then the area of the triangle formed  
 by  $L$  and the coordinate axes is 己.

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5. The solution of the integral equation

$$f(x) = 1999 + \int_0^x f(t) \cos t \, dt$$

is given by  $f(x) =$  庚.

6. The length of the parabolic spiral  $r = \theta^2$  ( $\theta \geq 0$ )

that lies inside the circle  $r = 4$  is 辛.

7. Suppose the temperature distribution of a ball centered at the origin is

$$T(x, y, z) = \frac{100}{1 + x^2 + y^2 + z^2}, \quad x^2 + y^2 + z^2 \leq 20.$$

Then the direction (which is a unit vector) of greatest

increase of temperature at the point  $(1, 2, 3)$  is 壬.

II. 計算與證明題 (共有兩大題, 每大題 14 分, 必需寫出

演算證明過程)

8. (14%)

Let  $\Omega = \{(x, y) | 0 \leq y \leq 1, x \geq y \text{ and } x^2 - y^2 \leq 1\}$ .

(a) Sketch the region  $\Omega$ .

(b) Evaluate the double integral

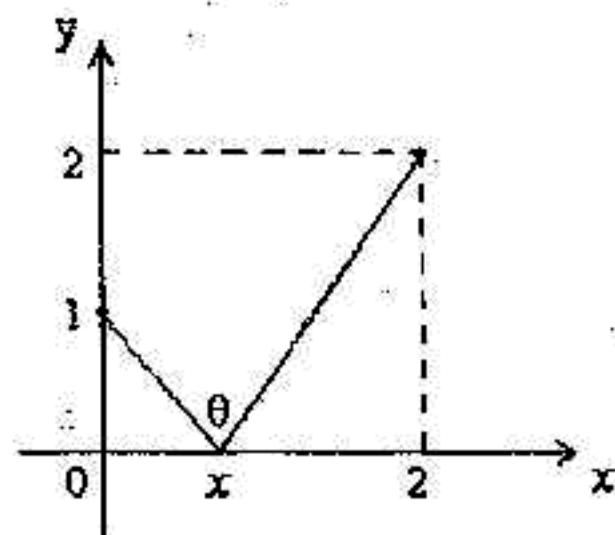
$$\iint_{\Omega} xy \sin(x^2 - y^2) \, dx \, dy.$$

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9. (14%) Consider the figure shown here:



(a) Show that  $\tan \theta = \frac{x+2}{x^2 - 2x + 2}$ .

(b) Find the value of  $x$  that maximizes the angle  $\theta$ .