

八十七學年度轉學生入學考試

科目 微積分(一般) 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

I. 填充題(共八題, 每題9分, 請將答案依甲, 乙, 丙, 一次序作答, 不需演算過程)

1. If $a_n = \frac{\left(\frac{10}{13}\right)^n}{\left(\frac{9}{16}\right)^n + \left(\frac{11}{12}\right)^n}$, then $\lim_{n \rightarrow \infty} a_n = \underline{\text{甲}}$.

2. Recall that $\sum_{k=0}^{\infty} \frac{1}{k!} = e$. Then $\sum_{k=0}^{\infty} \frac{k^2 + 3k}{(k+2)!} = \underline{\text{乙}}$.

3. A parallelogram has vertices at $A(2, -1, 4)$, $B(-1, 0, 5)$, $C(3, -2, 5)$ and D . The area of the orthogonal projection of the parallelogram onto the plane $x + y + z = 1$ is $\underline{\text{丙}}$.

4. If $x = t - t^2$, $y = t + t^2$, then $\frac{d^2y}{dx^2} \Big|_{t=1} = \underline{\text{丁}}$.

5. Suppose a, b are constants such that $\lim_{x \rightarrow 0} \left(\frac{\sin 2x}{x^3} + \frac{a}{x^2} + b \right) = 0$. Then $a + b = \underline{\text{戊}}$.

6. $\int_0^1 x \sin^{-1} x dx = \underline{\text{己}}$.

7. If $y = y(x)$ satisfies the differential equation $xy'' + y' = x$, $x > 0$ with initial value conditions $y(1) = \frac{1}{2}$, $y'(1) = 1$. Then $y(2) = \underline{\text{庚}}$.

8. Define $\max(a, b) = \begin{cases} a & \text{if } a \geq b \\ b & \text{if } b \geq a \end{cases}$. The value of $\int_0^1 \int_0^2 e^{\max(4x^2, y^2)} dy dx$ is $\underline{\text{辛}}$.

II. 計算與證明(必須寫出演算證明過程)

1. (9%)

Suppose $\sum_{n=1}^{\infty} a_n$, $\sum_{n=1}^{\infty} b_n$ are both convergent series with $a_n \geq 0, b_n \geq 0$

for all n . Prove that $\sum_{n=1}^{\infty} a_n b_n$ is also convergent.

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2. (9%)

Let $f(x) = a_1 \sin x + a_2 \sin 2x + \cdots + a_n \sin nx$ where a_1, \dots, a_n are real numbers and n is a positive integer. Suppose $|f(x)| \leq |x|$ for all real x . Prove that $|a_1 + 2a_2 + \cdots + na_n| \leq 1$.

3. (10%)

The temperature in a neighborhood of the point $(\frac{1}{4}\pi, 0)$ is given by the function $T(x, y) = \sqrt{2}e^{-y} \cos x$. Find the path followed by a heat-seeking particle that originates at $(\frac{1}{4}\pi, 0)$.