國 立 清 華 大 學 命 題 紙

八十七學年度轉學生入學考試

科目 微積分（經濟系） 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

I. 填充題（共八題，每題9分，請將答案依甲，乙，丙，…次序作答，不需演算過程）

1. If $a_n = \dfrac{\left(\frac{10}{11}\right)^n}{\left(\frac{9}{10}\right)^n + \left(\frac{11}{12}\right)^n}$, then $\lim_{n\to\infty} a_n =$ 甲 .

2. Recall that $\sum_{k=0}^{\infty} \frac{1}{k!} = e$. Then $\sum_{k=0}^{\infty} \frac{k^2+3k}{(k+2)!} =$ 乙 .

3. Let $\vec{A}$, $\vec{B}$, $\vec{C}$ be vectors in $\mathbb{R}^3$ such that $\vec{A}\cdot\vec{A} = 4$, $\vec{A}\cdot\vec{B} = 0$, $(\vec{A}\times\vec{B})\times\vec{C} = 0$, $(\vec{A}\times\vec{B})\cdot\vec{C} = 10$. Then $|\vec{B}\times\vec{C}| =$ 丙 .

4. The tangent line to the graph of $x^3 + y^3 = 2xy$ at $(1, 1)$ is 丁 .

5. Let a be a constant such that $\lim_{x\to 0}\left(\dfrac{\sin 3x}{x^2} + \dfrac{a}{x}\right) = 0$. Then $a =$ 戊 .

6. $\int_0^1 \frac{1}{1+\sqrt{x}}dx =$ 己 .

7. Let $G(t) = \int_0^t \int_0^x \sin(xy)dydx$. Then $G'\left(\sqrt{\frac{\pi}{2}}\right) =$ 庚 .

8. A housewife has a sum of money to deposit in a bank which gives her an interest compounded continuously at the rate of 6% annually. If she will get \$1000 forty months later, the sum that she is holding now is 辛 .

II. 計算與證明（必須寫出演算證明過程）

1. (10%)
Evaluate $\iint_\Omega |\sqrt{3}x - y|dxdy$ where Ω is the region in the first quadrant that lies inside the circle $x^2 + y^2 = 4$ and outside the circle $x^2 + y^2 = 1$.

2. (10%)
A rectangular box without a top is to have a volume of 12 cubic feet. Suppose that the material to be used to construct the box costs \$4.5 per square foot for the sides and \$4 per square foot for the bottom. Find the dimensions for the box that will yield the minimum cost.

3. (8%)
If $\sum_{n=1}^{\infty} a_n$ is a convergent series with $a_n \geq 0$ for all n, prove that $\sum_{n=1}^{\infty} a_n^2$ is also convergent.