

八十六學年度轉學生入學考試

科目 微積分(經濟) 共 1 頁第 1 頁 *請在試卷【答案卷】內作答

I、填充題(共九題,每題9分,請將答案依甲、乙、丙...次序作答,不需演算過程)

1. If $a_n = (1 + \frac{2}{n} + \frac{3}{n^2})^n$, then $\lim_{n \rightarrow \infty} a_n =$ 甲.

2. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \ln \sqrt[n]{1 + \frac{k}{n}} =$ 乙.

3. If a, b will maximize the value of the integral $\int_a^b (4 - x^2) dx$, then $b - a =$ 丙.

4. Let f be a twice differentiable function and let $u(x) = f(x^2 - 1)$. If $f'(0) = 1$, $f''(0) = 2$, then $u''(1) =$ 丁.

5. If $\frac{dx}{dt} = 1 - x$, then $\lim_{t \rightarrow \infty} x(t) =$ 戊.

6. The tangent plane to $x^2 + 2y^2 + z^2 = 7$ at $(1, 1, 2)$ is 己.

7. The disk $x^2 + y^2 \leq 1$ is revolved about the line $x + y = 2$ to generate a solid. Then the volume of the solid is 庚.

8. $\int_0^2 \frac{x+7}{x^2-x-6} dx =$ 辛.

9. $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} \sqrt{1-x^2-y^2} dy dx =$ 壬.

II、計算與證明(必須寫出演算證明過程)

1. (10%)

A rectangular wooden box with an open top is to contain 500 cm^3 . Ignoring the thickness of the wood, how is the box to be constructed so as to use the smallest amount of wood.

2. (9%)

Draw the graph of $f(x) = x^n e^{-x}$, $x \geq 0$, where n is a fixed positive integer.