國立清華大學命題紙

八十五學年度轉學生入學考試

科目 微積分(一般) 共 1 頁第 1 頁 *請在試卷【答案卷】內作答

I、填充題（共七題，每題10分，請依序作答）

1 Let $f(t) = \int_2^t \sqrt{\frac{4}{7} + u^5}\, du$, $F(x) = \int_1^{\sin x} f(t)\, dt$. Then $F''(\pi) =$ ___甲___.

2 Let $u(x_1, \cdots, x_n) = (x_1^2 + \cdots + x_n^2)^k$ where $n > 2$. If $\sum_{i=1}^{n} \frac{\partial^2 u}{\partial x_i^2} = 0$ for some $(x_1, \cdots, x_n) \neq (0, \cdots, 0)$ then $k =$ ___乙___.

3 If $y = y(x)$ satisfies the initial value problem $y'' - 3y' + 2y = 1 + e^x$, and $y(0) = 0, y'(0) = 0$. Then $y(-2) =$ ___丙___.

4 Find $\lim_{n\to\infty} [(n^{100} + n^{99})^{\frac{1}{100}} - n] =$ ___丁___.

5 Consider the line integral

$$\int_C (2x + 4x^3 y)dx + x^4 dy$$

where the path C is the line segment from $(0, \pi)$ to $(10, 1)$. The value of the integral is ___戊___.

6 Let S be the solid obtained by revolving the region $D = \{(x, y) | (x-2)^2 + y^2 \leq 1, y \geq 0\}$ around the line $y = x$. The volume of S is ___己___.

7 The curve $x^3 - y^3 = 1$ is asymptotically to the line $y = x$. The point on the curve $x^3 - y^3 = 1$ that is farthest to the line $y = x$ is ___庚___.

II、計算與証明(必須寫出演算証明過程)(每題10分)

1 Find $\int_0^{\pi} \sqrt{1 - \sin x}\, dx$.

2 If $a_0, a_1, \cdots, a_n$ are real numbers satisfying

$$\frac{a_0}{1} + \frac{a_1}{2} + \cdots + \frac{a_n}{n+1} = 0,$$

show that the equation $a_0 + a_1 x + \cdots + a_n x^n = 0$ has at least one real root.

3 The ideal gas law $PV = nRT$ (n is the number of moles of the gas, R is a constant) determines each of the three variables P, V, and T (pressure, volume and absolute temperature respectively) as the function of the other two. Show that

$$\frac{\partial P}{\partial V} \cdot \frac{\partial V}{\partial T} \cdot \frac{\partial T}{\partial P} = -1.$$