國 立 清 華 大 學 命 題 紙

八十五學年度轉學生入學考試

科目 微積分(經濟) 共 1 頁第 1 頁 *請在試卷【答案卷】內作答

I、填充題(共七題,每題10分,請依序作答)

1 The equation of the tangent line to the curve $2x^3 + 2y^3 - 9xy = 0$ at the point $(1, 2)$ is __甲__.

2 Let $f(t) = \int_2^t \sqrt{\frac{4}{7} + u^3}\, du$, $F(x) = \int_1^{\sin x} f(t)\, dt$. Then $F''(\pi) =$ __乙__.

3 Let $u(x_1, \cdots, x_n) = (x_1^2 + \cdots + x_n^2)^k$ where $n > 2$. If $\sum_{i=1}^{n} \frac{\partial^2 u}{\partial x_i^2} = 0$ for some $(x_1, \cdots, x_n) \neq (0, \cdots 0)$ then $k =$ __丙__.

4 If $y = y(x)$ satisfies the initial value problem

$$y'' - 3y' + 2y = 1 + e^{3x}, y(0) = 0, y'(0) = 0.$$

Then $y(x) =$ __丁__.

5 A hotel with 25 rooms normally charges 40 dollars for each room. However, special group rates are advertised: If a group reguires more than 6 rooms, the price for each room is decreased by 1 dollar times the number of rooms exceeding 6. The maximal revenue that the hotel can receive from a group is __戊__.

6 Find $\lim_{n\to\infty} \left[(n^{100} + n^{99})^{\frac{1}{100}} - n\right] =$ __己__.

7 Let R be the region defined by the intersection of the interiors enclosed by $r = 1$ and $r = 1 + \cos\theta$, $0 \leq \theta \leq 2\pi$. The area of R is __庚__.

II、計算與証明(必須寫出演算証明過程)(每題10分)

1 Find $\int_0^{\pi} \sqrt{1 - \sin x}\, dx$.

2 Find the points of the ellipse $x^2 + xy + y^2 = 3$ that are closest to and farthest from the origin.

3 If $a_0, a_1, \cdots, a_n$ are real numbers satisfying

$$\frac{a_0}{1} + \frac{a_1}{2} + \cdots + \frac{a_n}{n+1} = 0,$$

show that the equation $a_0 + a_1 x + \cdots + a_n x^n = 0$ has at least one real root.