1. Find orthogonal trajectories of the following curves. （計算題，應詳列計算過程，無計算過程者不予計分）。

    $y = cx$

    (15%)

2. Solve the following initial value problem. （計算題，應詳列計算過程，無計算過程者不予計分）。

    $$y'' + 9y = 15e^x, \qquad y(0) = 6, \ y'(0) = -2$$

    (15%)

3. Solve the following problem by Laplace transform. （計算題，請詳列計算過程，無計算過程者不予計分）。

    $$y'' + y' - 2y = 30u(t - \pi)\cos t, \qquad y(0) = \tfrac{1}{2}, \qquad y'(0) = -1$$

    (20%)

4. Find the Taylor series with center z_0 and its radius of convergence. （計算題，請詳列計算過程，無計算過程者不予計分）。

    $$\frac{1}{1+z}, \qquad z_0 = i$$

    (15%)

5. Find the corresponding Fourier series of the following periodic function. （計算題，請詳列計算過程，無計算過程者不予計分）。

    $$f(x) = x^2, \qquad -1 < x < 1, \qquad f(x+2) = f(x)$$

    (15%)

6. Let $\mathbf{u} = \begin{bmatrix} z & x & y \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} y+z & z+x & x+y \end{bmatrix}$, $f = x + y - z$, and $g = xyz$.

    Find the given expressions. （計算題，請詳列計算過程，無計算過程者不予計分）。

    a. $\nabla(fg)$ b. $\nabla \cdot (\mathbf{u} \times \mathbf{v})$

    c. $\nabla \times (g\mathbf{u})$ d. $\nabla \cdot (\nabla(fg))$

    （每小題 5%） (20%)