

科目 微積分 類組別 A2共 1 頁 第 1 頁

* 請在答案卷內作答

甲、填充題：共 8 題，每題 8 分，共 64 分。請在答案卷上列出題號依序作答。

請注意：本（甲、）部分，共 8 題，命題型態為填充題，不必詳列計算過程，倘若答案被包含在演算過程，將被視為試算流程，無法計分。

參考用

1. Find $\lim_{h \rightarrow 0} \frac{1}{h} \int_1^{1+h} \sqrt{1+t^2} dt$. Answer : _____

2. How many critical points does the function $f(x) = |x^2 - 1|$ have? Answer : _____

3. Find the number a and b such that $\lim_{x \rightarrow 0} \frac{\sqrt{ax+b}-2}{x} = 1$. Answer : _____

4. Market has two commodities A and B. The demand equations that relate the quantities demanded x and y to the unit prices p and q of the commodities A and B respectively are given by $x = f(p, q) = \frac{q}{p+q}$, $y = g(p, q) = e^{-(2q+p^2)}$. Are A and B substitute, complementary or neither? Answer : _____

5. Find the slope of the tangent line to the graph of the function $f(x) = (\tan x)^2 + e^x$ at $(0, 1)$. Answer : _____

6. Find the volume of solid bounded above by $z = f(x, y) = e^{-x^2}$ and below by the plane region R which is bounded by $y = x$, $x = 1$ and $y = 0$. Answer : _____

7. The Cobb-Douglas production function for a software manufacturer is given by $f(x, y) = 100x^{3/4}y^{1/4}$ where x represents the units of labor (at \$150 per unit) and y represents the units of capital (at \$250 per unit). The total cost of labor and capital is limited to \$50,000. Find x and y that will yield the maximum production level for this manufacturer.

Answer : _____

8. Suppose f'' is continuous on $[0, 1]$, $f(1) = 2$, $f'(1) = 2$ and the average value of f on $[0, 1]$ is 2. Evaluate $\int_0^1 x^2 f''(x) dx$. Answer : _____

乙、計算、證明題：共 3 題，每題 12 分，共 36 分。須詳細寫出計算及證明過程，否則不予計分。

1. Find any extrema of the function $f(x, y) = e^{-xy/4}$ subject to the constraint $x^2 + y^2 \leq 1$.

2. Evaluate $\int_0^6 \int_{x/3}^2 x \sqrt{y^3 + 1} dy dx$

3. Determine whether the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ converges absolutely or conditionally, or diverges.