科目 微積分 類組別 A2 A3 A4 A5 A6

＊請在試卷、答案卡內作答

甲、填充題：共 8 題，每題 8 分，共 64 分。請將答案依題號順序寫在答案卷上，不必寫演算過程。

1. Find the value of $\lim_{h\to 0}\dfrac{\cos(\pi+h)+1}{h}$.

Answer : _____

2. Find the area of the region bounded by the curves $y=\sin x$, $y=\cos x$, $x=0$, and $x=\pi/2$.

Answer : _____

3. Find the maximum rate of change of $f(x,y)=\sin(xy)$ at $(0,1)$.

Answer : _____

4. Let $g(x)$ be the inverse function of $f(x)=3+x^2+\tan(\pi x/2)$, $-1<x<1$. Find $g'(3)$.

Answer : _____

5. Let $f(x,y)=\begin{cases}\frac{x^2+2y^3}{x^2+y^2}, & \text{if } (x,y)\neq(0,0)\\ 0, & \text{if } (x,y)=(0,0).\end{cases}$ Find $f_y(0,0)$.

Answer : _____

6. Find the interval of convergence of the series $\sum_{n=2}^{\infty}(-1)^n\dfrac{x^n}{4^n\ln n}$.

Answer : _____

7. Convert the integral $\int_0^1\int_0^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} dzdydx$ to an equivalent integral in spherical coordinates.

Answer : _____ (**Do not evaluate the integral**).

8. Evaluate the iterated integral $\int_0^1\int_{\sqrt{y}}^1 \dfrac{ye^{x^2}}{x^3}\,dxdy$.

Answer : _____

乙、計算、證明題：共 3 題，每題 12 分，共 36 分。須詳細寫出計算及證明過程，否則不予計分。

1. Find the limit $\lim_{n\to\infty}\sum_{k=1}^{n}\dfrac{\sqrt{n^2-k^2}}{n^2}$.

2. Evaluate the line integral $\oint_C y^3\,dx-x^3\,dy$ where C is the circle $x^2+y^2=4$.

3. Find the extreme values of $f(x,y)=2x^2+3y^2-4x-5$ on the disk $x^2+y^2\le 16$.

參考用