注意：考試開始鈴響前，不得翻閱試題，
並不得書寫、畫記、作答。

國立清華大學 114 學年度碩士班考試入學試題

系所班組別：經濟學系

科目代碼：4803
考試科目：微積分與統計

一作答注意事項一

1. 請核對答案卷（卡）上之准考證號、科目名稱是否正確。

2. 考試開始後，請於作答前先翻閱整份試題，是否有污損或試題印刷不清，得舉手請監試人員處理，但不得要求解釋題意。

3. 考生限在答案卷上標記「▛ 由此開始作答」區內作答，且不可書寫姓名、准考證號或與作答無關之其他文字或符號。

4. 答案卷用盡不得要求加頁。

5. 答案卷可用任何書寫工具作答，惟為方便閱卷辨識，請儘量使用藍色或黑色書寫；答案卡限用 2B 鉛筆畫記；如畫記不清（含未依範例畫記）致光學閱讀機無法辨識答案者，其後果一律由考生自行負責。

6. 其他應考規則、違規處理及扣分方式，請自行詳閱准考證明上「國立清華大學試場規則及違規處理辦法」，無法因本試題封面作答注意事項中未列明而稱未知悉。

Part I: 微積分

1. **[10 pts]** Differentiate the following with respect to t:

$$t^n(a\sqrt{t} + b)$$

2. **[10 pts]** If $u(y)$ denotes an individual's utility of having consumption y, then $R = -yu''(y)/u'(y)$ is the coefficient of relative risk aversion. Assuming $y > 0$, please compute R for the utility function

$$u(y) = A_1 + \frac{A_2 y^{1-\rho}}{1-\rho},$$

where A_1, A_2, and ρ are positive constants with $\rho \neq 1$.

3. **[10 pts]** Prove that if u and v are differentiable functions of x and $u > 0$, then

$$y = u^v \longrightarrow y' = u^v(v'\ln u + \frac{vu'}{u})$$

4. **[10 pts]** Find the linear approximation to the following function about the point $u = 0$:

$$g(u) = A(1+u)^{\frac{a}{1+b}} - 1$$

(A, a, b are positive constants)

5. **[10 pts]** Calculate $\int_0^{1/3} \frac{dx}{e^x + 1}$

Part II: 統計

[**Instructions**: Please do all questions and show your work in details.]

1. [**20 pts**] A sealed bid auction has an economist for sale to the highest of n bidders. You are bidder #1. Your experience is that the bids of each other bidder is distributed with a Power distribution $F(X) = X^\alpha$ for $0 \leq X \leq 1$. Your profit if you are successful in buying the economist at price y is $1 - y$, while the profit if losing is zero.

   (a) What is the probability that you win the bid?

   (b) What should you bid to maximize your expected profit?

2. [**30 pts**] Suppose the following linear regression model:

$$Y_i = \beta X_i + \varepsilon_i, \quad i = 1, ..., n \tag{1}$$

   where β denotes regression coefficient, and ε_i is the random error term with mean zero and $E[\varepsilon_i^2] = \sigma^2$.

   (a) What is the variance of the random error term?

   (b) Please write down the normal equation(s)?

   (c) Suppose that ε_i is observable. What is the unbiased estimator of σ^2?

   (d) Please derive the ordinary least squares (OLS) estimator of β.

   (e) The OLS residual is denoted by e_i. What is $e_1 + e_2 + ... + e_n$?

   (f) The method of moment estimator based on $E[\varepsilon_i] = 0$, say $\tilde{\beta}$, is as efficient as the OLS estimator $\hat{\beta}$. Please comment.