

注意：考試開始鈴響前，不得翻閱試題，
並不得書寫、畫記、作答。


國立清華大學 113 學年度碩士班考試入學試題

系所班組別：經濟學系

科目代碼：4603

考試科目：微積分與統計

—作答注意事項—

1. 請核對答案卷（卡）上之准考證號、科目名稱是否正確。
2. 考試開始後，請於作答前先翻閱整份試題，是否有污損或試題印刷不清，得舉手請監試人員處理，但不得要求解釋題意。
3. 考生限在答案卷上標記「由此開始作答」區內作答，且不可書寫姓名、准考證號或與作答無關之其他文字或符號。
4. 答案卷用盡不得要求加頁。
5. 答案卷可用任何書寫工具作答，惟為方便閱卷辨識，請儘量使用藍色或黑色書寫；答案卡限用 2B 鉛筆畫記；如畫記不清（含未依範例畫記）致光學閱讀機無法辨識答案者，其後果一律由考生自行負責。
6. 其他應考規則、違規處理及扣分方式，請自行詳閱准考證明上「國立清華大學試場規則及違規處理辦法」，無法因本試題封面作答注意事項中未列明而稱未知悉。

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共 3 頁，第 1 頁 *請在【答案卷、卡】作答

Part I

請在答案卷作答，只以最後答案正確與否計分，計算過程不計分。

1. (6 分) Find the longest distance from the curve $x^3 - xy + y^3 = 1$ ($x \geq 0, y \geq 0$) to the origin.
2. (6 分) Solve the differential equation $y'' - 2y' + y = e^x$.
3. (6 分) Solve the differential equation $y' - 2xy = 5xy^2$.
4. (6 分) Compute the indefinite integral $\int \frac{-2x}{x^3 + x^2 + x + 1} dx$.
5. (6 分) Compute the indefinite integral $\int e^{2x} \sin(3x) dx$.
6. (7 分) Compute the indefinite integral $\int \frac{dx}{\sin x (\sin x + \cos x)}$.
7. (6 分) Compute the indefinite integral $\int 2x^2(4x + 1)^{\frac{5}{2}} dx$.
8. (7 分) Evaluate $\int_0^1 f(x) dx$ where $f(x) = x \int_1^x \frac{\sin(t^2)}{t} dt$.

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共 3 頁，第 2 頁

*請在【答案卷、卡】作答

Part II

1. (15 points) The time between arrivals of patients at the emergency room of a hospital follows an exponential probability distribution with a mean of 15 minutes, so that the probability density function is:

$$f(x) = \frac{1}{15} e^{-\frac{x}{15}}, x \geq 0$$

where $e = 2.71828$.

- (a) (5 points) What is the standard deviation of x ?
- (b) (5 points) What is the probability that the arrival time between patients is above 20 minutes?
- (c) (5 points) What is the probability that the arrival time between patients is between 10 and 20 minutes?

2. (20 points) A sample of size n is randomly drawn from a population with mean μ and variance σ^2 .

- (a) (6 points) Sample mean $\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$ is a commonly used estimator of the population mean (μ). An economist proposes to use an alternative estimator

$$\tilde{x} = \frac{2}{n} \left(\frac{1}{4} x_1 + \frac{3}{4} x_2 + \cdots + \frac{1}{4} x_{n-1} + \frac{3}{4} x_n \right), \text{ where the sample size } n \text{ is an even number.}$$

Is \tilde{x} an unbiased estimator of the population mean? Explain why or why not.

- (b) (7 points) Calculate $\text{Var}(\tilde{x})$. Is \tilde{x} more or less efficient than \bar{x} ?

- (c) (7 points) This economist also proposes to use $\hat{\sigma}^2 = \frac{\sum_{i=1}^n (x_i - \tilde{x})^2}{n}$ as an estimator of the population variance σ^2 . Is $\hat{\sigma}^2$ an unbiased estimator? Explain why or why not.

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共 3 頁，第 3 頁 *請在【答案卷、卡】作答

3. (15 points) An economist is studying individuals' decision to participate in the labor force. Let LF be a dummy variable for whether an individual participates in the labor force (LF = 1) or not (LF = 0). Let edu and age be the years of education and age respectively. Let male be a dummy variable that indicate if the individual is male (male = 1) or female (male = 0). Suppose that, using data on 200 individuals, the following logit model is obtained:

$$\hat{P}(LF = 1|edu, age, male) = \frac{e^{(-1+0.03edu+0.05age-0.0005age^2+0.8male)}}{1 + e^{(-1+0.03edu+0.05age-0.0005age^2+0.8male)}}$$

- (a) (4 points) The economist includes both age and age squared (age²) in the equation. Based on the coefficient estimates, how would you interpret the relation between age and probability of participating in the labor force?
- (b) (6 points) What is the probability of labor force participation for male individuals that are 40 years old with 12 years of education? What is the estimated odds in favor of labor market participation?
- (c) (5 points) What is the estimated odds ratio of male relative to female?