注意：考試開始鈴響前，不得翻閱試題，

並不得書寫、畫記、作答。

國立清華大學 113 學年度碩士班考試入學試題

系所班組別：數學系

科 目 代 碼 ：0101

考 試 科 目 ：高等微積分

—作答注意事項—

1. 請核對答案卷（卡）上之准考證號、科目名稱是否正確。
2. 考試開始後，請於作答前先翻閱整份試題，是否有污損或試題印刷不清，得舉手請監試人員處理，但不得要求解釋題意。
3. 考生限在答案卷上標記「由此開始作答」區內作答，且不可書寫姓名、准考證號或與作答無關之其他文字或符號。
4. 答案卷用盡不得要求加頁。
5. 答案卷可用任何書寫工具作答，惟為方便閱卷辨識，請儘量使用藍色或黑色書寫；答案卡限用 2B 鉛筆畫記；如畫記不清（含未依範例畫記）致光學閱讀機無法辨識答案者，其後果一律由考生自行負責。
6. 其他應考規則、違規處理及扣分方式，請自行詳閱准考證明上「國立清華大學試場規則及違規處理辦法」，無法因本試題封面作答注意事項中未列明而稱未知悉。

國立清華大學 113 學年度碩士班考試入學試題

系所班組別：數學系碩士班

考試科目（代碼）：高等微積分（0101）

　　*請在【答案卷】作答

You need to explain how you get your answers in each problem. Please mark your answers clearly.

Problem 1: (10 points) Let $P = x - y^3 - y$ and $Q = x^3 + y^2$. Find the line integral of $\oint_C Pdx + Qdy$ where C is the simple closed curve $x^2 + y^2 = 1$. The line integral is taken counter-clockwisely.

Problem 2: (10 points) Find $\int_2^{24} \frac{d[\sqrt{x}]}{\sqrt{x}}$ where $[\cdot]$ is the greatest integer function.

Problem 3: Let $f_n(x) = nx^n(1-x)$, $n = 1, 2, 3, \cdots$.
(a) (5 points) Determine if the sequence $\{f_n\}_{n=1}^{\infty}$ converges pointwisely on $[0,1]$.
(b) (10 points) Determine if the family of functions $\{f_n\}_{n=1}^{\infty}$ is equicontinuous on $[0,1]$.

Problem 4: (10 points) Let $f(x,y) = \frac{x^5 - y^3}{x^4 + y^2}$ when $(x,y) \neq (0,0)$ and $f(0,0) = 0$. Determine if f is differentiable at $(0,0)$.

Problem 5: (15 points) Let $S := \{(x, sin(\frac{1}{x}))|\ 0 < x < \pi\} \cup \{(0,y)|\ 0 \leq y \leq 1\ , y \in \mathbb{Q}\}$. Determine if S is connected. You can use the fact that $\{(x, sin(\frac{1}{x}))|\ 0 < x < \pi\}$ is connected.

Problem 6: (15 points) Let f be an Riemann integrable function on $[0,1]$. Suppose that for any $0 < a < b < 1$, there is at least one number $c \in (a,b)$ such that $f(c) = 1$ or $f(c) = 0$. If $\int_0^1 f(x)dx = 1/2$, prove that there is an uncountable subset $S \subseteq [0,1]$ such that $f(s) = 0$ for any $s \in S$. Note that f may not be continuous.

Problem 7: (10 points) Prove that $\sqrt[3]{x}$ is uniformly continuous on $\mathbb{R}$.

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Problem 8: (15points) Let $s_n(x) = sinx + \frac{1}{2}sin2x + \frac{1}{3}sin3x + \cdots + \frac{1}{n}sinnx$. Prove that the sequence $\{s_n\}_{n=1}^{\infty}$ converges uniformly on $[1, 2]$.