注意：考試開始鈴響前，不得翻閱試題，
並不得書寫、畫記、作答。

國立清華大學 114 學年度碩士班考試入學試題

系所班組別：工程與系統科學系
乙組

科 目 代 碼 ：3201

考 試 科 目 ：工程數學

—作答注意事項—

1. 請核對答案卷（卡）上之准考證號、科目名稱是否正確。
2. 考試開始後，請於作答前先翻閱整份試題，是否有污損或試題印刷不清，得舉手請監試人員處理，但不得要求解釋題意。
3. 考生限在答案卷上標記「由此開始作答」區內作答，且不可書寫姓名、准考證號或與作答無關之其他文字或符號。
4. 答案卷用盡不得要求加頁。
5. 答案卷可用任何書寫工具作答，惟為方便閱卷辨識，請儘量使用藍色或黑色書寫；答案卡限用 2B 鉛筆畫記；如畫記不清（含未依範例畫記）致光學閱讀機無法辨識答案者，其後果一律由考生自行負責。
6. 其他應考規則、違規處理及扣分方式，請自行詳閱准考證明上「國立清華大學試場規則及違規處理辦法」，無法因本試題封面作答注意事項中未列明而稱未知悉。

國立清華大學 114 學年度碩士班考試入學試題

系所班組別：工程與系統科學系碩士班 乙組(0532)

考試科目（代碼）：工程數學 (3201)

1. Find the solution of the given differential equation. Your answer should include the general solution $y(x)$ and the exact solution when an initial or boundary condition is provided.

   (a) $\frac{dy}{dx} + \frac{y}{x} = e^{x}$. (5%)

   (b) $\frac{dy}{dx} = \frac{-x}{x^2y+2y}$. (5%)

   (c) $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = 8x,\ y(1) = 0$ and $y'(1) = 0$ (5%)

2. Find the series solution of the following differential equation about x = 0.

   $$3x\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0. \qquad (10\%)$$

   You have to express the solution in the form of $y(x) = C_1y_1(x) + C_2y_2(x)$. To save time, you can only show the first four terms of $y_1(x)$ and $y_2(x)$.

3. Use the Laplace transform to solve the problem and obtain $y(t)$.

   $$\frac{dy}{dt} + 3y + 2\int_0^t y(\tau)d\tau = u(t-1) - u(t-2), \qquad (10\%)$$

   where $y(0) = 0$ and $u(t)$ is the unit step function.

4. Consider the matrix

   $$M = \begin{bmatrix} 0 & 4 & -2 \\ -1 & 2 & 1 \\ -4 & 4 & 2 \end{bmatrix}.$$

   (a) Find the determinant of M and obtain the inverse matrix M^{-1}. (7%)

   (b) Estimate the eigenvalues and eigenvectors of M. (8%)

5. (a) Evaluate the surface integral $\oiint_S \mathbf{F}\cdot\mathbf{n}dA$ where $\mathbf{F} = (x^3 - y^3, y^3 - z^3, z^3 - x^3)$ and S is the surface of $x^2 + y^2 + z^2 \le 30,\ z \ge 0$. (5%)

(b) Calculate the line integral $\oint_C \mathbf{F}\cdot\mathbf{r}'ds$ by Stokes' theorem for $\mathbf{F}=(z^3,x^3,y^3)$ where C is a loop defined by $x=2, y^2+z^2=16$, oriented counterclockwise with respect to the x-axis. (5%)

(c) Use Cauchy's residue theorem to determine $\oint_C \frac{\exp(z)-\exp(2z)}{z^2(z+1)}dz,\ C: z(t)=3\cos t+i3\sin t, 0\le t\le 2\pi.$ (5%)

(d) Find the Laurent series of the complex function $f(z)=(z+2)\cos\left(\frac{1}{z+1}\right), |z+1|>0.$ (5%)

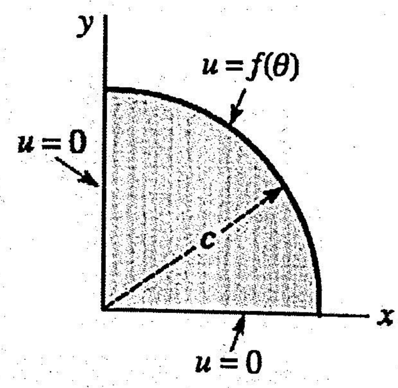
6. The boundary value problem is $\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}=0,$

$0<r<c,\ 0<\theta<\frac{\pi}{2}$ with

$u(r,0)=u\left(r,\frac{\pi}{2}\right)=0$ for $0<r<c,$

$u(c,\theta)=f(\theta)$ for $0<\theta<\frac{\pi}{2}.$

Find the solution $u(r,\theta)$. (10%)



7. Use Laplace transform to solve the heat equation $u_{xx}=u_t, x>0, t>0$, subject to the condition: $u(0,t)=\begin{cases}30, & 0<t<1\\ 0, & t\ge 1\end{cases}$, $u(x,0)=50,\ \lim_{x\to\infty}u(x,t)=50.$

Hint: The Laplace transform of $\operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right)$ is $\frac{e^{-a\sqrt{s}}}{s}$. (10%)

8. Use Fourier integral transform to solve the wave equation: $a^2\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial t^2}, -\infty<x<\infty, t>0,$ with $u(x,0)=f(x)$ and $u_t(x,0)=0$ for $-\infty<x<\infty$. (10%)