注意：考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。

國立清華大學 115 學年度碩士班考試入學試題

系所班組別：動力機械工程學系
乙組(電機控制組)

科 目 代 碼 ：1302

考 試 科 目 ：控制系統

—作答注意事項—

1. 請核對答案卷（卡）上之准考證號、科目名稱是否正確。
2. 考試開始後，請於作答前先翻閱整份試題，是否有污損或試題印刷不清，得舉手請監試人員處理，但不得要求解釋題意。
3. 考生限在答案卷上標記「由此開始作答」區內作答，且不可書寫姓名、准考證號或與作答無關之其他文字或符號。
4. 答案卷用盡不得要求加頁。
5. 答案卷可用任何書寫工具作答，惟為方便閱卷辨識，請儘量使用藍色或黑色書寫；答案卡限用 2B 鉛筆畫記；如畫記不清（含未依範例畫記）致光學閱讀機無法辨識答案者，其後果一律由考生自行負責。
6. 提早交卷者不可攜出試題，請將試題連同答案卷(卡)交予監考人員，攜出將以違規論處。當節考試結束後試題將置於講台，可自行取回，本試場不負保管責任。
7. 其他應考規則、違規處理及扣分方式，請自行詳閱准考證明上「**國立清華大學試場規則及違規處理辦法**」，無法因本試題封面作答注意事項中未列明而稱未知悉。

國立清華大學 115 學年度碩士班考試入學試題

系所班組別：動力機械工程學系碩士班　乙組(電機控制組)

考試科目（代碼）：控制系統(1302)

　　*請在【答案卷　】作答

Q1. In this problem we will design a lead-compensator for an unstable plant $G(s) = \frac{K}{s^2-1}, (K > 0)$ step by step.

(a) Plot the bode plots of $G(s)$ and $G'(s) = \frac{K}{(s+1)^2}, (K > 0)$. Identify the corner frequencies of the two systems. (4 pts)

(b) Plot the root locus of the compensated unity feedback system $\frac{G}{1+G}$ as K varies. Explain whether such a control system is appropriate. (4 pts)

(c) Now we consider adding a lead controller so that the loop transfer function becomes $\frac{K}{s^2-1}\frac{1+aTs}{1+Ts}$ where a, T and K are the control parameters to be determined. Assume that the gain crossover frequency occurs at $\omega_c = 2\, rad/s$ where the loop transfer function exhibits the minimum phase lag, and that the corresponding phase margin is 60°. Determine the numerical values of a, T and K. (12 pts)

(d) To verify the stability of the control system you design in (c), plot the Nyquist plot of the corresponding loop transfer function. Using the Nyquist criterion to examine the closed-loop stability. What is the gain margin? (10 pts)

Q2. For the DC motor given in Fig. 1. Assume that the motor has a torque constant K_i and a back emf constant K_b and its rotor inertia is J_m. Ignore the rotational damping, frictional effect, and the loading torque (T_L).

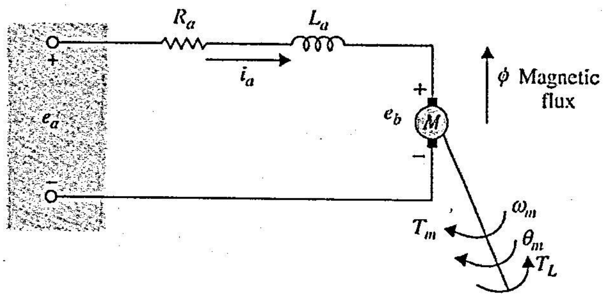

Fig. 1

(a) (Voltage control) Derive the transfer function from input voltage e_a to the rotor angle θ_m. (7 pts)

(b) (Current control) Derive the transfer function from input current i_a to the rotor angle θ_m. (6 pts)

(c) Assume that the motor is connected to the op-amp circuitry shown in Fig. 2. The objective is to make the motor current i_a track the input voltage V_{in}, thereby enabling implementation of the current control scheme described in part (b). Derive the transfer function from $V_{in}(s)$ to $i_a(s)$. (7 pts)

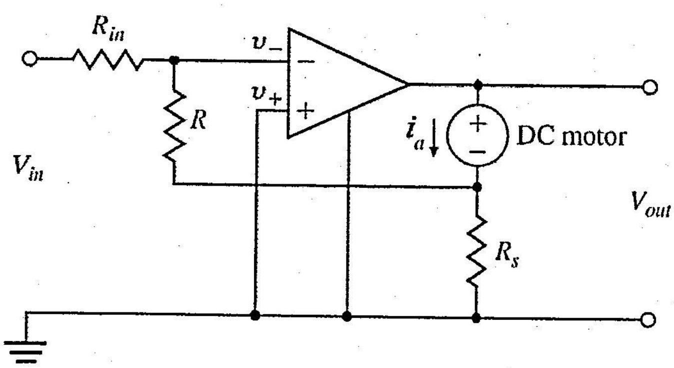

Fig. 2

Q3. Let's consider the saturation element shown in Fig. 3 and the small-signal situation where the signal reach is always below the saturation. The slope of the saturation element is K.

(a) Compute the roots of the closed-loop characteristic equation when the slope K is 1. (5 pts)

(b) For (a), determine (i) the system type (5 pts) and (ii) the corresponding steady-state error with respect to reference inputs(5 pts).

(Notes: (i) counts only if (ii) is correct.)

(c) Before any internal signal reach saturation, (i) sketch the root locus with respect to the slope K (K>0) (10 pts) and (ii) give the break-away/break-in points (5 pts).

(d) For (c), what maximum value of slope K is required to stabilize the closed-loop system? (5 pts)

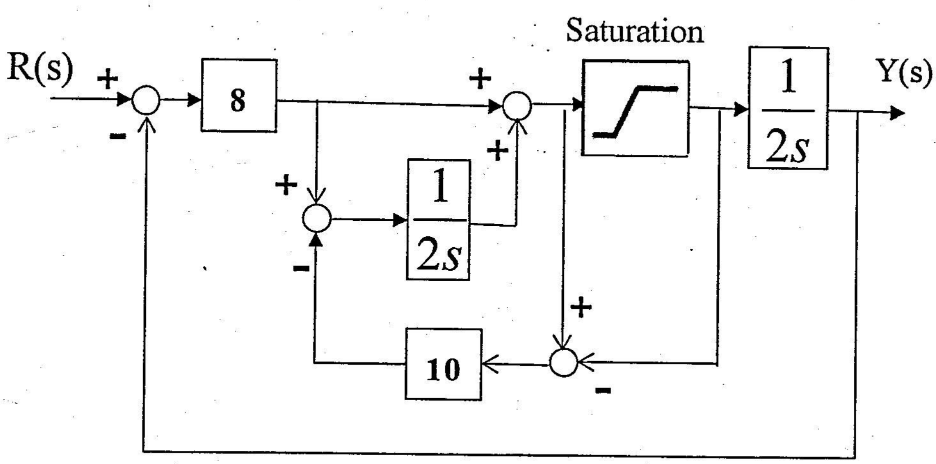

Fig. 3: Block diagram illustrating integral anti-windup

Q4. The equation of motion for a motor control in state space representation is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w;$$ (w is the process noise)

$$y = \begin{bmatrix} 0 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \nu$$ (ν is the sensor noise)

$$u = -\begin{bmatrix} k_1 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Here the performance cost function is $J = \int_0^\infty [\rho y^2(t) + u^2(t)]dt$

Use Linear Quadratic Regulator (Symmetric Root Locus) to find the optimal control ($[k_1 \quad k_2]$), which minimizes the cost function J, and gives the closed-loop system with the settling time of 1.15 seconds.

(a) What is the targeting closed-loop poles? (5 pts)

(b) what is the optimal control ($[k_1 \quad k_2]$)? (10 pts)