國立清華大學命題紙

99學年度 計量財務金融學系乙組(財務工程組) 碩士班入學考試

科目 微積分 科目代碼 4604 共 1 頁，第 1 頁 *請在【答案卷卡】作答

Total 100 points.

1. (25 points)
   (1) Let x be a real number. If $f(x)$ is a real-valued function and is continuous at $x = x_0$, state the $\epsilon - \delta$ definition $\lim_{x \to x_0} f(x)$?
   (2) Within the interval $[0, \pi/2]$, what would continuous point(s) be for the function $f(x) = \sin(x)$ when x is a rational number and $f(x) = 0$ when x is an irrational number?
   (3) Use the $\epsilon - \delta$ definition to prove the limit obtained in (c).

2. (30 points)
   (1) Find the derivative of $(\frac{1}{y} - \frac{1}{y^3}) \exp(-\frac{y^2}{2})$.
   (2) Use the Fundamental Theorem of Calculus to prove that
   $$\int_x^\infty \exp(-\frac{y^2}{2}) dy \geq (\frac{1}{x} - \frac{1}{x^3}) \exp(-\frac{x^2}{2}) \text{ for } x > 0.$$
   (3) For $y > x > 0$, use $\exp(-\frac{y^2}{2}) \leq \frac{y}{x} \exp(-\frac{y^2}{2})$ to prove that
   $$\int_x^\infty \exp(-\frac{y^2}{2}) dy \leq \frac{1}{x} \exp(-\frac{x^2}{2}). \quad (1)$$

3. (25 points)
Given two real-valued functions f and g, we say that $f(x) = O(g(x))$ as x approaches infinity iff $lim_{x \to \infty} |\frac{f(x)}{g(x)}| \leq M$, for some constant M.
   (1) What is the geometric meaning of $f(x) = O(g(x))$? (Simply draw a graph to illustrate the idea.)
   (2) Prove that $x \ln x = O(x^{1+p})$ for any $0 < p < 1$.
   (3) Use results from Equation (1) to prove that
   $$\int_x^\infty \exp(-\frac{y^2}{2}) dy = O(\frac{1}{x} \exp(-\frac{x^2}{2})).$$

4. (20 points)
Consider the following one-dimensional differential equation: for $t \geq 0$,
$$\frac{dX(t)}{dt} = \alpha(m - X(t)) + \sigma \frac{dg(t)}{dt},$$
with the initial condition $X(0) = m$ and $g(t)$ is some differentiable function. Use procedures (1) assume $Y(t) = m - X(t)$ and (2) differentiate $e^{\alpha t} Y(t)$ to solve this equation.