國立清華大學 105 學年度碩士班考試入學試題

系所班組別：計量財務金融學系碩士班 甲組、乙組

考試科目（代碼）：統計學 (4303)(4403)

　　*請在【答案卷、卡】作答

請依序作答，否則後果自行負責。只需寫下答案與簡短解釋，計算過程不需附上。

1. (10%) Four mice are chosen (without replacement) from a litter containing two white mice. The probability that both white mice are chosen is twice the probability that neither is chosen. How many mice are there in the litter?

2. (10%) Suppose that $Z_i \sim N(0,\sigma^2)$ and Z_i are independent. Please find the limiting distribution of

$$\sum_{i=1}^{n}\left(Z_i + \frac{\sigma^2}{n}\right) / \sqrt{n}.$$

3. (10%) Let X and Y be independent and normally distributed random variables with the same PDF

$$\frac{1}{\sqrt{2\pi}} e^{-x^2/2}.$$

Find the PDFs of $X^2 + Y^2$. (Please write down the formula of the PDF)

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4. (10%) Assume that the return R_t of a stock has the following log-normal distribution for fixed t:

$$\log(R_t) \sim N(\mu, \sigma^2).$$

Suppose we let the density of $\log(R_t)$ be denoted by $f(R_t)$ and hypothesize that $\mu = 0.17, \sigma^2 = 0.09$. Find a function $\xi(R_t)$ such that under the density, $f(R_t)\,\xi(R_t)$, R_t has a mean equal to the risk-free rate $r_f = 0.05$.

5. (10%) Following the above question, find a $\xi(R_t)$ such that R_t has a mean zero.

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6. (10%) Brendan Dennehy works for Transon Investments, Plc., a Dublin-based hedge fund with significant equity investments in technology companies in Asia, North America, and Europe. Transon is concerned by the recent poor performance of one of the fund's Chinese investments, Winston Communications, an assembler of telecommunications equipment. Transon's chief of information technology (IT) is Sean Malloy. Yesterday, Winston's IT office sent Malloy data relating to the assembly process and a printout of an analysis of the number of defective assemblies per hour. Winston's IT people believe that the number of defective assemblies per hour (D_t) is a function of the outside air temperature and the speed (production rate) of the assembly lines. Malloy recalls that Dennehy had substantial training in statistics while working on his MBA. He asks Dennehy to help him interpret the regression results in Exhibit 1 and 2 supplied by Winston.

Exhibit 1: Regression Results: $D_t = b_0 + b_1 AIR_t + b_2 R_t + \varepsilon_t$

Regression Results	Coefficient	Std. Error
Constant (b_0)	0.016	0.0942
AIR_t : Outside air temperature (b_1)	0.0006	0.001
R_t : Assembly line speed (b_2)	0.5984	0.3
Number of observations used in the regression	384	
Critical t value at 5% significance (two-tail test)	1.96	
R Square	0.414	
Std. Error of the Estimate	0.333	
Durbin-Watson	1.89	
F test	157.699	
Significance of F test	0	
Durbin-Watson critical values (5% significance)	(1.63, 1.72)	
Correlation between outside air temperature and assembly line speed	0.015	

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Exhibit 2: Results of the Dickey-Fuller Tests

Time Series	Coefficient	Std. Error
Defective assemblies per hour	0.0036	0.0023
Outside air temperature	-0.423	0.0724
Assembly line speed	-0.586	0.043

Test critical values of Dickey-Fuller test for *t*-statistic are:
1% level = -3.4534, 5% level = -2.8716, 10% level = -2.5722
The null hypothesis of this test is: has unit root.

What is the *most* appropriate inference from the Durbin–Watson statistic reported in Exhibit 1? The Durbin–Watson test:

(A) is inconclusive.

(B) rejects the null hypothesis of no positive serial correlation.

(C) fails to reject the null hypothesis of no positive serial correlation.

(D) none of the above.

7. (10%) Assuming a 5% level of significance, the *most* appropriate conclusion that can be drawn from the Dickey–Fuller results reported in Exhibit 2 is that the:

(A) test for a unit root is inconclusive for the dependent variable.

(B) independent variables exhibit unit roots but the dependent variable does not.

(C) dependent variable exhibits a unit root but the independent variables do not.

(D) none of the above.

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8. (10%) The table below summarizes observed data on the gender and party membership of 1000 individuals:

Gender	Democrat	Republican	Total
Male	300	300	600
Female	200	200	400
Total	500	500	1000

Which one of the following statements about the relationship between gender and party is suggested by the data in the table?

(A) There is a relationship between gender and party membership.

(B) There is a relationship between gender and being a Democrat but not between gender and being a Republican.

(C) There is no relationship between gender and party membership.

(D) There is a relationship between gender and party membership for males but not for females.

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9. (10%) Suppose a 95% confidence interval for p, the proportion of drivers who admit that they sometimes run red lights when no one is around, is 0.29 to 0.38. Which of the following statements is FALSE?

(A) A test of H_0: $p=0.3$ versus H_1: $p\neq 0.3$ would not be rejected using significant level $\alpha=0.05$.

(B) A test of H_0: $p=0.5$ versus H_1: $p\neq 0.5$ would be rejected using $\alpha=0.05$.

(C) It is plausible that about 37% of all drivers would admit that they sometimes run red lights when no one is around.

(D) It is plausible that a majority of all drivers would admit that they sometimes run red lights when no one is around.

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10. (10%) Which of the following statements are not true?

(A) In some experiments, different samples contain different numbers of observations. However, the concepts and methods of single-factor ANOVA are most easily developed for the case of equal sample sizes.

(B) The population or treatment distributions in single-factor ANOVA are all assumed to be normally distributed with the same variance σ^2.

(C) In one-way ANOVA, if either the normality assumption or the assumption of equal variances is judged implausible, a method of analysis other than the usual F test must be employed.

(D) The test statistic for single-factor ANOVA is F = MSA/MSE, where MSA is the mean square for treatments, and MSE is the mean square for error.

(E) All of the above statements are not true.

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Standard normal cumulative distribution function $\Phi(z)$ and $100 \times \gamma$th percentiles z_γ

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8314	0.3340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

γ	0.90	0.95	0.975	0.99	0.995	0.999	0.9995	0.99995	0.999995
z_γ	1.282	1.645	1.960	2.326	2.576	3.090	3.291	3.891	4.417

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100 × γth Percentiles $t_\gamma(v)$ of Student's *t* distribution with *v* degrees of freedom

$$\gamma = \int_{-\infty}^{t_\gamma(v)} f(t;\, v)\, dt$$

	γ								
v	0.60	0.70	0.80	0.90	0.95	0.975	0.99	0.995	0.9995
1	0.325	0.727	1.376	3.078	6.314	12.706	31.821	63.657	636.619
2	0.289	0.617	1.061	1.886	2.920	4.303	6.965	9.925	31.598
3	0.277	0.584	0.978	1.638	2.353	3.182	4.541	5.841	12.924
4	0.271	0.569	0.941	1.533	2.132	2.776	3.747	4.604	8.610
5	0.267	0.559	0.920	1.476	2.015	2.571	3.365	4.032	6.869
6	0.265	0.553	0.906	1.440	1.943	2.447	3.143	3.707	5.959
7	0.263	0.549	0.896	1.415	1.895	2.365	2.998	3.499	5.408
8	0.262	0.546	0.889	1.397	1.860	2.306	2.896	3.355	5.041
9	0.261	0.543	0.883	1.383	1.833	2.262	2.821	3.250	4.781
10	0.260	0.542	0.879	1.372	1.812	2.228	2.764	3.169	4.587
11	0.260	0.540	0.876	1.363	1.796	2.201	2.718	3.106	4.437
12	0.259	0.539	0.873	1.356	1.782	2.179	2.681	3.055	4.318
13	0.259	0.538	0.870	1.350	1.771	2.160	2.650	3.012	4.221
14	0.258	0.537	0.868	1.345	1.761	2.145	2.624	2.977	4.140
15	0.258	0.536	0.866	1.341	1.753	2.131	2.602	2.947	4.073
16	0.258	0.535	0.865	1.337	1.746	2.120	2.583	2.921	4.015
17	0.257	0.534	0.863	1.333	1.740	2.110	2.567	2.898	3.965
18	0.257	0.534	0.862	1.330	1.734	2.101	2.552	2.878	3.922
19	0.257	0.533	0.861	1.328	1.729	2.093	2.539	2.861	3.883
20	0.257	0.533	0.860	1.325	1.725	2.086	2.528	2.845	3.850
21	0.257	0.532	0.859	1.323	1.721	2.080	2.518	2.831	3.819
22	0.256	0.532	0.858	1.321	1.717	2.074	2.508	2.819	3.792
23	0.256	0.532	0.858	1.319	1.714	2.069	2.500	2.807	3.767
24	0.256	0.531	0.857	1.318	1.711	2.064	2.492	2.797	3.745
25	0.256	0.531	0.856	1.316	1.708	2.060	2.485	2.787	3.725
26	0.256	0.531	0.856	1.315	1.706	2.056	2.479	2.779	3.707
27	0.256	0.531	0.855	1.314	1.703	2.052	2.473	2.771	3.690
28	0.256	0.530	0.855	1.313	1.701	2.048	2.467	2.763	3.674
29	0.256	0.530	0.854	1.311	1.699	2.045	2.462	2.756	3.659
30	0.256	0.530	0.854	1.310	1.697	2.042	2.457	2.750	3.646
40	0.255	0.529	0.851	1.303	1.684	2.021	2.423	2.704	3.551
60	0.254	0.527	0.848	1.296	1.671	2.000	2.390	2.660	3.460
120	0.254	0.526	0.845	1.289	1.658	1.980	2.358	2.617	3.373
∞	0.253	0.524	0.842	1.282	1.645	1.960	2.326	2.576	3.291

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100 × γth Percentiles $\chi^2_\gamma(\nu)$ of the chi-square distribution with ν degrees of freedom

$$\gamma = \int_0^{\chi^2_\gamma(\nu)} h(y;\nu)\,dy$$

	γ													
ν	0.005	0.010	0.025	0.050	0.100	0.250	0.500	0.750	0.900	0.950	0.975	0.990	0.995	0.999
1					0.02	0.10	0.45	1.32	2.71	3.84	5.02	6.63	7.88	10.83
2	0.01	0.02	0.05	0.10	0.21	0.58	1.39	2.77	4.61	5.99	7.38	9.21	10.60	13.82
3	0.07	0.11	0.22	0.35	0.58	1.21	2.37	4.11	6.25	7.81	9.35	11.34	12.84	16.27
4	0.21	0.30	0.48	0.71	1.06	1.92	3.36	5.39	7.78	9.49	11.14	13.28	14.86	18.47
5	0.41	0.55	0.83	1.15	1.61	2.67	4.35	6.63	9.24	11.07	12.83	15.09	16.75	20.52
6	0.68	0.87	1.24	1.64	2.20	3.45	5.35	7.84	10.64	12.59	14.45	16.81	18.55	22.46
7	0.99	1.24	1.69	2.17	2.83	4.25	6.35	9.04	12.02	14.07	16.01	18.48	20.28	24.32
8	1.34	1.65	2.18	2.73	3.49	5.07	7.34	10.22	13.36	15.51	17.53	20.09	21.96	26.12
9	1.73	2.09	2.70	3.33	4.17	5.90	8.34	11.39	14.68	16.92	19.02	21.67	23.59	27.88
10	2.16	2.56	3.25	3.94	4.87	6.74	9.34	12.55	15.99	18.31	20.48	23.21	25.19	29.59
11	2.60	3.05	3.82	4.57	5.58	7.58	10.34	13.70	17.28	19.68	21.92	24.72	26.76	31.26
12	3.07	3.57	4.40	5.23	6.30	8.44	11.34	14.85	18.55	21.03	23.34	26.22	28.30	32.91
13	3.57	4.11	5.01	5.89	7.04	9.30	12.34	15.98	19.81	22.36	24.74	27.69	29.82	34.53
14	4.07	4.66	5.63	6.57	7.79	10.17	13.34	17.12	21.06	23.68	26.12	29.14	31.32	36.12
15	4.60	5.23	6.26	7.26	8.55	11.04	14.34	18.25	22.31	25.00	27.49	30.58	32.80	37.70
16	5.14	5.81	6.91	7.96	9.31	11.91	15.34	19.37	23.54	26.30	28.85	32.00	34.27	39.25
17	5.70	6.41	7.56	8.67	10.09	12.79	16.34	20.49	24.77	27.59	30.19	33.41	35.73	40.79
18	6.26	7.01	8.23	9.39	10.86	13.68	17.34	21.60	25.99	28.87	31.53	34.81	37.16	42.31
19	6.84	7.63	8.91	10.12	11.65	14.56	18.34	22.72	27.20	30.14	32.85	36.19	35.58	43.82
20	7.43	8.26	9.59	10.85	12.44	15.45	19.34	23.83	28.41	31.41	34.17	37.57	40.00	45.32
21	8.03	8.90	10.28	11.59	13.24	16.34	20.34	24.93	29.62	32.67	35.48	38.93	41.40	46.80
22	8.64	9.54	10.98	12.34	14.04	17.24	21.34	26.04	30.81	33.92	36.78	40.29	42.80	48.27
23	9.26	10.20	11.69	13.09	14.85	18.14	22.34	27.14	32.01	35.17	38.08	41.64	44.18	49.73
24	9.89	10.86	12.40	13.85	15.66	19.04	23.34	28.24	33.20	36.42	39.36	42.98	45.56	51.18
25	10.52	11.52	13.12	14.61	16.47	19.94	24.34	29.34	34.38	37.65	40.65	44.31	46.93	52.62
30	13.79	14.95	16.79	18.49	20.60	24.48	29.34	34.80	40.26	43.77	46.98	50.89	53.67	59.70
40	20.71	22.16	24.43	26.51	29.05	33.66	39.34	45.62	51.80	55.76	59.34	63.69	66.77	73.40
50	27.99	29.71	32.36	34.76	37.69	42.94	49.33	56.33	63.17	67.50	71.42	76.15	79.49	86.66
60	35.53	37.48	40.48	43.19	46.46	52.29	59.33	66.98	74.40	79.08	83.30	88.38	91.95	99.61
70	43.28	45.44	48.76	51.74	55.33	61.70	69.33	77.58	85.53	90.53	95.02	100.42	104.22	112.32
80	51.17	53.54	57.15	60.39	64.28	71.14	79.33	88.13	96.58	101.88	106.63	112.33	116.32	124.84
90	59.20	61.75	65.65	69.13	73.29	80.62	89.33	98.64	107.56	113.14	118.14	124.12	128.30	137.21
100	67.33	70.06	74.22	77.93	82.36	90.13	99.33	109.14	118.50	124.34	129.56	135.81	140.17	149.45

For large ν, $\chi^2_\gamma(\nu) \doteq \nu[1 - (2/9\nu) + z_\gamma\sqrt{(2/9\nu)}]^3$.