(Problem 1 and 2) Two triangular-shaped unidentified flying objects (UFO) in the sky are both observed moving at a speed of $\sqrt{0.75c}$. The UFO1 is moving along $+x$-direction and the UFO2 is moving along $+y$-direction with respect to a witness standing on the ground as shown in Figure 1. The witness claimed that the two objects seemingly to be identical in size and are both equilateral triangles with all 6 sides seems to have length $a$. Suddenly, UFO2 changes its direction (without rotation) and moves along $-x$-direction with velocity $\sqrt{0.75c}$ with respect to the witness frame as shown in Figure 2.

1. What’s the area ratio of UFO2 and UFO1 in the witness frame after the UFO2 changes its direction?
   
   (A) $\frac{3}{2}$  (B) $\sqrt{3}$  (C) $\frac{11}{2}$  (D) $2\sqrt{3}$  (E) $\frac{2}{\sqrt{3}}$

2. What’s the observed speed of UFO2 in UFO1 reference frame after the UFO2 changes its direction?
   
   (A) $\frac{\sqrt{6}}{5}c$  (B) $\frac{\sqrt{3}}{5}c$  (C) $\frac{\sqrt{6}}{7}c$  (D) $\frac{\sqrt{3}}{2}c$  (E) $\frac{4\sqrt{3}}{7}c$

3. For a one-dimensional quantum harmonic oscillator with potential energy $U(x) = (1/2)m\omega^2 x^2$, (A) the ground state wave function is $\psi_0 \propto \exp(-m\omega x^2/\hbar)$. (B) the position uncertainty of the ground state is maximum. (C) the ground state energy is $(\hbar/2\pi)\omega$  (D) the momentum uncertainty for all states are the same (E) none of the above.

4. Which of the following phenomena is not due to the tunneling effect? (A) field emission microscope (B) Lamb shift (C) inversion of ammonia molecule (D) imaging of surface atomic contour with sharp conductive point (E) $\alpha$ decay of a radioactive nucleus.

5. For hydrogen atom, which of the following is not correct? (A) The total energy is determined by the principle quantum number. (B) All s-states are spherically symmetric. (C) The $n$th energy level is $n^2$ degenerate. (D) It is possible to specify simultaneously any two components of angular momentum. (E) The angular dependence of the wave function is solely determined by the spherical harmonic functions.
6. About electron spin, which of the following is not correct? (A) The Pauli exclusion principle prohibits two electrons to possess the same spin quantum number. (B) The sodium doublet is due to the spin-orbital interaction. (C) The Stern-Gerlach experiment leads to the discovery of electron spin. (D) Electron is a Fermion because its wave function is antisymmetric in the coordinate. (E) Electron is a Fermion because its spin is 1/2.

7. About atomic emission, which of the following is not correct? (A) The non-zero spectral linewidth reflects that the lifetime of occupancy is finite. (B) Selection rules prohibit certain transitions to occur. (C) The origin of the selection rules is that the angular momentum has to be conserved between the emitted photon and the electron. (D) Stimulated emission emits "cloned" photon of the original photon. (E) None of the above.

8. Consider two independent spin-zero bosons – there is no interaction between them, and we do not need to worry about a spin label – each in the same one-dimensional infinite well. The single particle wave function in that case is \( u_n(x) = \sqrt{\frac{2}{L}} \sin \left( \frac{n \pi x}{L} \right) \), where \( n \) is the quantum number that labels the state. Which one is the wave function of the two boson system? (\( N' \) denotes the normalization constant.)

(A) 0
(B) \( 2N' \frac{2}{L} \sin \left( \frac{n \pi x}{L} \right) \sin \left( \frac{n' \pi x}{L} \right) \)
(C) \( N' \left( \frac{2}{L} \sin \left( \frac{n \pi x}{L} \right) \cos \left( \frac{n' \pi x}{L} \right) + \frac{2}{L} \cos \left( \frac{n \pi x}{L} \right) \sin \left( \frac{n' \pi x}{L} \right) \) \)
(D) \( 2N' \frac{2}{L} \sin \left( \frac{n \pi x}{L} \right) \cos \left( \frac{n' \pi x}{L} \right) \)
(E) \( N' \left( \frac{2}{L} \sin \left( \frac{n \pi x}{L} \right) \cos \left( \frac{n' \pi x}{L} \right) - \frac{2}{L} \cos \left( \frac{n \pi x}{L} \right) \sin \left( \frac{n' \pi x}{L} \right) \) \)

9. Consider the \( n = 2, \ell = 1 \) energy states of hydrogen. The gas is placed in a constant magnetic field of strength 6 Tesla. Estimate the ratio of the splitting of levels to the original energy of the states.

(A) 1.0 × 10^-4  (B) 4.7 × 10^-2  (C) 7.2 × 10^-4  (D) 2.0 × 10^-3  (E) 6.5 × 10^-3

Problem 10, 11 and 12) A scientist is setting up an apparatus for photocathode effect as shown in Figure 3. He prepared a piece of metal as the target. When the incident light wavelength is 207 nm with power 1 mW, the obtained stopping/retarding voltage is 2 V and the maximum current obtained can reach \( I_0 \). Figure 3
10. What the $I-V$ plot(s) may be if the scientist uses a 1 mW laser as the light source which emits 103.5 nm light?

11. If the scientist uses a constant power (1 mW) laser which wavelength can be tuned from 414 nm to 150 nm, what the possible $I-V$ plot(s) should be for any single wavelength in the tuning range.

12. How’s the stopping voltage related to the incident light? ($f$, $\lambda$, and $E_{hv}$ stand for the incident light frequency, wavelength, and energy, respectively.)
13. A non-relativistic electron is confined in a cubical box which has size $a$ in all three dimensions. What's the lowest possible kinetic energy according to the uncertainty principle? (A) $\frac{\hbar}{4a \pi m_e}$, (B) $\frac{3\hbar^2}{64a^3 \pi^3 m_e}$, (C) $\left( \frac{\hbar}{4am_e} \right)^2$, (D) $\frac{3\hbar^2}{8a^3 m_e}$, (E) $\frac{\hbar^2}{64a^3 \pi^3 m_e}$.

14. Which of the following statements is/are not correct regarding the generation of X-ray in the tube?

(A) The minimum electron matter wavelength is $\sqrt{\left( \frac{\hbar}{\lambda_{ev}} \right)^2 - (m_e c)^2}$.

(B) The electrons should be treated as relativistic particles.

(C) The minimum electron matter wavelength is $\frac{\hbar \lambda_{ev}}{2m_e c}$.

(D) If the target is replaced by an element with doubled atomic number, the shortest generated X-ray wavelength will be $\frac{\lambda_{ev}}{2}$.

(E) The X-ray spectrum from the X-ray tube has maximum output photon number at $\lambda_{ev}$.

15. Which of the following statements is/are true regarding the positron in this experiment?

(A) After the collision the total energy of the positron is $\frac{\sqrt{\frac{2E\hbar^2}{\lambda}}}{\hbar}$.

(B) The collision is elastic.

(C) The photon wavelength ratio between the incident and scattered photon is $\frac{\hbar}{m_i c \lambda_{ev}}$.

(D) The positron has to be treated as relativistic particle after collision.

(E) If the incident X-ray wavelength becomes $\frac{\lambda_{ev}}{2}$, the change of photon frequency before and after the collision is about $1.24 \times 10^{10}$ Hz.
16. Which of the following statements are true? (A) Classical electrons obey the law of Fermi-Dirac distribution. (B) The Fermi level (i.e., the highest occupied level) at 0 K of a system of free classical electrons is above the lowest energy level. (C) Photons obey the law of Bose-Einstein distribution with a vanishing chemical potential. (D) Photons at extremely high temperatures obey approximately the law of Boltzmann distribution. (E) All of the above.

17. Consider the molecule of H₂. Which of the following statements are true? (A) The binding energy is of the order of meV. (B) The binding energy is of the order of atomic level spacing. (C) The binding energy is of the order of eV. (D) The molecule is likely to dissociate into atoms at a temperature of the order 1000 K. (E) The molecule is likely to dissociate into atoms at a temperature of the order 10000 K.

18. Consider the copper metal. Which of the following statements are true? (A) There are energy gaps which are of the order eV in the energy spectrum of electrons. (B) Pure copper is a good electrical conductor. (C) Pure copper is not a good heat conductor. (D) The copper ions are tightly bound by covalent bonds. (E) Pure copper is a superconductor at cryogenic temperature.

19. Consider the elemental semiconductor Si. Which of the following statements are true? (A) The Si ions are tightly bound by ionic bonds. (B) Pure Si is a good electrical conductor at room temperature. (C) Pure Si can easily be bent. (D) The energy gap between the conduction and valence bands is of the order eV. (E) Pure Si at a temperature of 10000 K, if not melting, would be a good electrical conductor.

20. Consider the (substitutionally) doped Si. Which of the following statements are true? (A) If As atoms are implanted in Si, it is p-type. (B) If Ga atoms are implanted in Si, it is p-type. (C) If Ge atoms are implanted in Si, it is p-type. (D) Doped Si is a perfect electrical conductor with vanishing resistance. (E) None of the above.

Fundamental Constants:

1. Electron charge: \( e = 1.6 \times 10^{-19} \text{ C} \)
2. Reduced Planck's constant: \( \hbar = 1.05 \times 10^{-34} \text{ J s} \)
3. Speed of light: \( c = 3.0 \times 10^8 \text{ m/s} \)
4. Electron mass: \( m_e = 9.1 \times 10^{-31} \text{ Kg} \)
5. Boltzmann's constant: \( k_B = 1.38 \times 10^{-23} \text{ J/K} \)