1. Solve the following ordinary differential equations

   (i) $y'' = x (y')^3$  
   (10%)

   (ii) $x^2 y'' + (x - 1)(x y' - y) = x^2 e^{-x}$  
   (10%)

2. Find the inverse Laplace transform of

   $$F(s) = \frac{e^{-s}}{s(s+1)(s+2)}$$  
   (10%)

3. Given the matrix

   $$A = \begin{bmatrix} 31 & 9\sqrt{3} \\ 40 & 40 \\ 9\sqrt{3} & 13 \\ 40 & 40 \end{bmatrix},$$

   compute $\lim_{n \to \infty} A^n$  
   (10%)

4. Evaluate the line integral of the normal derivative of a function $w(x,y)$ counterclockwise over the boundary curve $C$ of the rectangle defined by $0 \leq x \leq 1$, and $0 \leq y \leq 2$, i.e. to evaluate the following integral

   $$\oint_C \frac{\partial w}{\partial n} \, ds \quad \text{with} \quad w = e^x + e^{2y}$$  
   (10%)
5. Any periodic function \( x(t) \), of period \( 2a \), can be expressed in the form of a complex Fourier series \( x(t) = \sum_{n=-\infty}^{\infty} c_n e^{i \omega_n t} \) where \( \omega_n \) is the fundamental frequency given as \( \omega_n = \frac{\pi}{a} \). Using the relation \( \int e^{kt} dt = \frac{e^{kt}}{k^2} (kt - 1) \) to find the complex Fourier series expansion of the function below,

\[
x(t) = \begin{cases} 
  A(1 + \frac{t}{a}), & -a \leq t \leq 0 \\
  A(1 - \frac{t}{a}), & 0 \leq t \leq a 
\end{cases}
\]

with the period \( 2a \) and the fundamental frequency \( \omega_n \). (15%)