1. Use the Laplace transform to solve the given initial-value problem

\[ y'''' - 3y''' + 3y'' - y = t^2 e^t, \quad y(0) = 1, y'(0) = 0, y''(0) = -2 \] (20%)

2. Suppose A, B, and C are all matrices, indicate true or false for each of the following statements. (No proof is needed.)

(a) If AB=0 then A=0 or B=0;
(b) If A^2=0 then A=0;
(c) (A+B)(A-B)=A^2 - B^2;
(d) If AB=AC and A is non-invertible, then B=C;
(e) All the eigenvalues of a n×n Hermitian matrix are real;
(f) If A is a n×n unitary matrix and its eigenvalue is \( \lambda \), then |det(A)|=1 and |\( \lambda \)|=1. (6%)

3. A bar of length L is perfectly insulated both at the ends x=0 and x=L. The initial temperature distribution at the bar is \( u(x, 0) = f(x) \). Physical information: the flux of heat through the faces at the ends is proportional to the values of \( \frac{\partial u}{\partial x} \) there. The governing equation is \( \frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \).

(a) Show the temperature distribution \( u(x, t) \) within the bar using separation of variables. (15%) 
(b) If \( f(x) = 1 + \cos(2\pi x / L) \), what are the final temperatures at x=0, L/2 and L as \( t \to \infty \)? (5%)
4. (a) Given a vector \( \vec{A} = A_x \vec{a}_x + A_y \vec{a}_y + A_z \vec{a}_z \), where \( \vec{a}_x, \vec{a}_y, \vec{a}_z \) are the unit vectors in \( x, y, z \) directions, respectively. Express \( \text{Curl} \ \vec{A} \ (\nabla \times \vec{A}) \) and \( \text{Div} \ \vec{A} \ (\nabla \cdot \vec{A}) \) in terms of \( A_x, A_y \) and \( A_z \). (6%)  
(b) Given a vector function \( \vec{F} = y\vec{a}_x + x\vec{a}_y \), evaluate the scalar line integral \( \int_{\vec{l}} \vec{F} \cdot d\vec{l} \) from \( P_1(2,1,-1) \) to \( P_2(8,2,-1) \) along the straight line joining the two points. (14%)  

5. (a) Find the inverse Laplace transform of the function  
\[
F(s) = \frac{s^2 - 5s + 4}{s(s^2 + 1)}
\]  
(b) Find the Laplace transform of the function  
\[
g(t) = \begin{cases} 
2t; & t < 3 \\
1; & t \geq 3 
\end{cases}
\]  

(10%)  

6. Please use contour integration to evaluate the integral \( \int_0^\infty \frac{x^2}{1+x^4} \, dx \). (14%)