1. Solve the following initial value problem
   \[ y' = 1 + y^2, \quad y(0) = 0 \] (15%)

2. Solve the following initial value problem
   \[ y'' + y' = 2 + 2x + x^2, \quad y(0) = 8, \quad y'(0) = -1 \] (15%)

3. Find eigenvalues and eigenfunctions of the following problem
   \[ (x^{-1}y')' + (\lambda + 1)x^3y = 0, \quad y(1) = 0, \quad y(e) = 0 \] (15%)

4. Using Laplace transforms, solve the following integral equation
   \[ y = 2t - 4 \int_0^t y(\tau)(t - \tau) d\tau \] (15%)

5. Diagonalize the following matrix.
   \[
   \begin{bmatrix}
   5 & 10 & -10 \\
   10 & 5 & -20 \\
   5 & -5 & -10 \\
   \end{bmatrix}
   \] (15%)

6. Evaluate the following surface integral \[ \iint_S F \cdot ndA \], where
   \[ F = [x = z, y + z, x + y] \] and \( S \) is the sphere of \( x^2 + y^2 + z^2 = 9 \) (10%)

7. Find the complex Fourier series of the following function
   \[ f(x) = x^3, \quad -\pi < x < \pi \] (10%)

8. Find solution \( u(x, y) \) of the following equation
   \[ xu_{xy} + 2yu = 0 \] (10%)