甲、填空題：共 10 題，每題 8 分，共 80 分。請將答案依題號順序寫在答案卷上，不必寫演算過程。

1. Find the value of a such that \( \lim_{x \to 0} \frac{\sqrt{ax+8} - 2}{x} = \frac{5}{12} \). Answer: 

2. The coordinates of a particle in the \( xy \)-plane are differentiable functions of time \( t \) with \( dx/dt = -1 \) m/sec and \( dy/dt = -5 \) m/sec. How fast is particle's distance form the origin changing as it passes through the point \((5, 12)\)? Answer: 

3. What is the value of the definite integral \( \int_{0}^{1} \frac{x}{\sqrt[4]{4} + 5x} \, dx \)? Answer: 

4. Find \( f(4) \) if \( \int_{0}^{x} f(t) \, dt = x \cos \pi x \). Answer: 

5. Find the length of the curve \( y = \int_{0}^{x} \sqrt{\cos 2t} \, dt \) for \( 0 \leq x \leq \pi/4 \). Answer: 

6. Evaluate \( \int_{1}^{\infty} \frac{\ln x}{x^2} \, dx \). Answer: 

7. Find a value for the constant \( b \) that will make the radius of convergence of the power series \( \sum_{n=2}^{\infty} \frac{b^n x^n}{\ln n} \) equal to 5. Answer: 

8. Find the value of \( a \) that make \( f(x, y) = \begin{cases} \sin xy & \text{if } xy \neq 0 \\ xy & \text{if } xy = 0 \end{cases} \) continuous on \( \mathbb{R}^2 \). Answer: 

9. Find the area of the region \( R \) in the \( xy \)-plane enclosed by the circle \( x^2 + y^2 + 4 \), above the line \( y = 1 \), and below the line \( y = \sqrt{3}x \). Answer: 

10. Let \( f(x, y) = x^2 - xy + y^2 - 3y \). Find the direction \( u \) for which \( D_u f(1, -1) = 4 \). Answer: 

乙、計算、證明題：共 2 題，每題 10 分，共 20 分。須詳細寫出計算及證明過程，否則不予計分。

1. Find the highest and lowest points on the curve \( x^2 + xy + y^2 = 12 \).

2. (a) If \( F(x, y) = x^2 i + y^2 j \), find a function \( f \) such that \( F = \nabla f \).  

(b) Evaluate the line integral \( \int_C F \cdot dr \), where \( C \) is the arc of the parabola \( y = 2x^2 \) from \((-1, 2)\) to \((2, 8)\).