

九十二學年度 科技管理研究所 系(所) 甲 組碩士班研究生招生考試

科目 微積分 科號 5701 共 1 頁第 1 頁 \*請在試卷【答案卷】內作答

I. 填充題(共八題,每題9分請將答案依甲,乙,丙,……,次序作答,不須演算過程)

1. If  $\lim_{x \rightarrow 0} \frac{\sin 2x + ax + bx^3}{x^3} = 2$  then  $a + b =$  甲.
2. A circle of radius  $\sqrt{2}$  with center  $(0, a)$  is inscribed (內切於) in the parabola  $y = 2x^2$ . Then  $a =$  乙.
3. Suppose  $H(x)$  is a differentiable function on  $(0, \infty)$  and satisfies  $H(x^2) = \frac{1}{x^2} \int_3^{x^2} [2t^2 - H'(t)] dt$ . Then  $H'(3) =$  丙.
4.  $\int_0^1 \frac{dx}{\sqrt[3]{x+\sqrt{x}}} =$  丁.
5. Let  $S$  be the solid bounded above by the cone  $z = 2 - \sqrt{x^2 + y^2}$  and bounded below by the disk:  $(x - 1)^2 + y^2 \leq 1$ . Then the volume of  $S =$  戊.
6.  $\sum_{n=1}^{\infty} \frac{1}{n!(n+2)} =$  己. (Hint: Think of  $xe^x$ )
7. If the least amount of paper per volume is required to make a conical (圓錐形) paper cup then the ratio  $h/r$  of the height  $h$  and the radius  $r$  of the rim is 庚.
8. Let  $P$  be the path of steepest descent along the surface  $z = x^2 + 3y^2$  from the point  $(1, -2, 13)$ . Then the projection curve onto the  $xy$ -plane of  $P$  is 辛.

II. 計算與證明(必須寫出演算證明過程)

- 1.(10%) Prove that  $\sum_{n=1}^{\infty} (-1)^n \left[ \left(1 + \frac{1}{n}\right)^{n+1} - \left(1 + \frac{1}{n}\right)^n \right]$  is convergent.
- 2.(10%) A company needs a warehouse to contain  $10^6$  ft<sup>3</sup>. They estimate that the floor and ceiling of the building will cost NT\$3 per square foot to construct and the walls will cost NT\$7 per square foot. Find the cost of the most economical rectangular building.
- 3.(8%) Suppose that  $f(x, y) = 0$  implicitly defines  $y$  as a function of  $x$  and that  $y''$  and the second partial derivatives of  $f$  exist. Express  $y''$  in terms of the first and second partial derivatives of  $f$ .