(1). (10 points) Calculate the following integrals

$$\text{(i)} \int_0^\infty \frac{e^x}{1+e^{2x}}\,dx, \qquad \text{(ii)} \int_0^{\frac{1}{2}} \frac{dx}{1-x^4}.$$

(2). (10 points) Find the following limits

$$\text{(i)} \lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{\ln(1+x)}\right), \qquad \text{(ii)} \lim_{x\to 0}\left(\frac{1+2^x}{2}\right)^{\frac{1}{x}}.$$

(3). (10 points) Determine the convergence or divergence of the following series, and give your reason.

$$\text{(i)} \sum_{k=1}^\infty (\sqrt{k}-\sqrt{k-1})^k, \qquad \text{(ii)} \sum_{k=1}^\infty \frac{k!}{k^k}.$$

(4). (10 points) Find the area of the surface generated by revolving the curve $y = \cos x$, $x \in [0, \frac{\pi}{2}]$, about the x-axis.

(5). (10 points) Let C be the curve given by $x^{\frac{1}{2}} + y^{\frac{1}{2}} = 3$, $x > 0$, $y > 0$. Suppose $P = (a, b)$ is a point on C. Find the sum of the x-intercept and the y-intercept of the tangent line to C through P and show that this value is independent of the point P.

(6). (10 points) Is the following function

$$f(x,y) = \begin{cases} \frac{x^2 y}{x^4 + y^2}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0), \end{cases}$$

differentiable at (0,0)? Show your reasons.

(7). (10 points) A rectangular box is to be inscribed in the cone

$$z = 9 - \sqrt{x^2+y^2}, \quad z \geq 0.$$

Find the dimensions(size and volume) for the box that maximize its volume.

(8). (10 points) Evaluate the line integral counterclockwise along the curve $C = \{(x,y) \mid x^2+y^2 = 1\}$,

$$\int_C (2xy + \sin x)dx + (\tan^{-1}(1+y^2) + x^3 + y)dy.$$

(9). (10 points) Solve the differential equation on $\mathbb{R}^2$,

$$(3x^2\cos y)dx + (2y - x^3\sin y)dy = 0.$$

(10). (10 points) Evaluate the surface integral

$$\iint_S z d\sigma,$$

where S is the hemisphere $z = 1 + \sqrt{1 - x^2 - y^2}$ with $x^2 + y^2 \leq 1$ and $d\sigma$ is the surface element on S.