國立清華大學106學年度碩士班考試入學試題

系所班組別：科技管理研究所碩士班

考試科目(代碼)：統計學(4302)

共 2 頁，第 1 頁　　*請在【答案卷、卡】作答

1. (a) Give me an example to illustrate the importance of statistics. (10%)

   (b) How could you apply what statistics knowledge you learned before to the technology management? (10%)

2. (a) Why do we need the probability density function and probability mass function? What is the difference between them? (10%)

   (b) What is the moment-generating function and why do we need it? (5%) Why do we call $E(e^{tX})$ as the moment-generating function? (5%)

3. (a) Show that if the random variable X is $N(\mu, \sigma^2), 0 < \sigma^2 < \infty$, then the random variable $\nu = \left(\frac{X-\mu}{\sigma}\right)^2 = Z^2$ is $\chi^2(1)$. (5%)

   (b) Let $X_1, X_2, \ldots, X_n$ be observations of a random sample of size n from the normal distribution $N(\mu, \sigma^2)$ .Then show that the sample mean, $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$, and the sample variance, $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$,are independent and $\frac{(n-1)S^2}{\sigma^2} = \frac{\sum_{i=1}^{n}\sum_{i=1}^{n}(X_i-\bar{X})^2}{\sigma^2} \sim \chi^2(n-1)$.(5%)

4. (a) A random variable X has an exponential distribution if its probability density function is defined by

   $f(x) = \frac{1}{\theta}e^{-\frac{x}{\theta}}, 0 \leq \theta < \infty, \theta > 0,$

   what are the mean and variance of X? (6%)

   (b) If $\theta = 20$, what is the probability that X is less than 18? (4%)

5. (a) What is the central limit theorem? (5%)

   (b) The servicing times for customers coming through a checkout counter in a retail store are independent random variables with mean 1.5 minutes and variance 1. Approximate the probability that 100 customers can be served in less than 2 hours of total service time. (5%)

6. Let $Y_1, Y_2, \ldots, Y_n$ be a random sample from a normal distribution with mean μ and variance σ^2. Find the MLEs of μ and σ^2.Both MLEs of μ and σ^2 are unbiased? (10%)

7. Suppose that Y_1 , Y_2 ,..., Y_n denote a random sample from a Poisson distribution with mean λ. (i) Find the MLE $\hat{\lambda}$ for λ. (ii) Find the expected value and variance of $\hat{\lambda}$. (iii) Show that the estimator of part (a) is consistent for λ. (iv) What is the MLE for $P(Y = 0) = e^{-\lambda}$. (10%)

8. (a) A company produces machined engine parts that are supposed to have a diameter variance no larger than .0002 (diameters measured in inches). A random sample of ten parts gave a sample variance of .0003. Test, at the 5% level, $H_0: \sigma^2 = .0002$ against $H_a: \sigma^2 > .0002$. (5%)

   (b) Suppose that we wish to compare the variation in diameters of parts produced by the company in part (a) with the variation in diameters of parts produced by a competitor. Recall that the sample variance for our company, based on $n = 10$ diameters, was $s_1^2 = .0003$. In contrast, the sample variance of the diameter measurement for 20 of the competitor's parts was $s_2^2 = .0001$. Do the data provide sufficient information to indicate a smaller variation in diameters for the competitor? Test with $\alpha = .05$. (5%)

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補充 1 頁　　*請在【答案卷、卡】作答

$\chi^2_{0.05}$

df	
5	11.0705
6	12.5916
7	14.0671
8	15.5073
9	16.9190.

F test

Denominator df	α	Numerator df 8	Numerator df 9
19	0.100	2.02	1.98
	0.050	2.48	2.42.
	0.010	3.63	3.52.