國 立 清 華 大 學 命 題 紙

99 學年度 統計學研究所 碩士班入學考試 *請在【答案卷卡】作答

科目 基礎數學(微積分、線性代數) 科目代碼 0101 共 3 頁第 1 頁

※ 作答時，請將答案統一寫在答案卷的第一頁，並清楚標示題號。請使用第二頁以後之試卷來解題，解題的計算過程不列入評分。

1. (6%) Determine the set of points at which the function

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+xy+y^2}, & \text{if } (x,y) \neq 0 \\ 0, & \text{if } (x,y) = 0 \end{cases}$$

is continuous.

2. Evaluate each of the following limits.

(5%) (a)

$$\lim_{x\to\infty}(1+2x)^{1/3x}$$

(5%) (b)

$$\lim_{x\to 0^+}(\sin(x))^{1/\ln(x)}$$

3. (6%) Find the length L of the curve with parametric equations: $x=t^2$, $y=2t$, $z=\ln(t)$ from point (1,2,0) to $(e^2,2e,1)$, where

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2}\,dt.$$

4. Find what values of x do the following series converge?

(6%) (a)

$$\sum_{n=1}^{\infty}\frac{x^{n-1}}{n\cdot 3^n}$$

(6%)(b)

$$\sum_{n=1}^{\infty}\frac{n(x-1)^n}{2^n(3n-1)}$$

5. (6%) Evaluate

$$\iint_R \sqrt{x^2+y^2}\,dxdy,$$

where **R** is the region in the xy plane bounded by $x^2+y^2=4$ and $x^2+y^2=9$.

6. Let

$S_n(x) = nxe^{-nx^2}, n = 1,2,3,\cdots, 0 \le x \le 1.$

(6%) (a) Determine whether

$$\lim_{n\to\infty}\int_0^1 S_n(x)dx = \int_0^1 \lim_{n\to\infty} S_n(x)dx.$$

(6%) (b) Explain the result in (a). (Less than 20 words.)

7. (6%) Find the maximum value of $x^2+y^2+z^2$ subject to the constraint conditions $x^2/4+y^2/5+z^2/25=1$ and $z=x+y$.

8. (6%) Find an equation of the tangent plane to the surface

$$z = e^{x^2-y^2}$$

at the point $(x_0, y_0, z_0)=(1,-1,1)$.

9. Determine whether

$$\lim_{m\to\infty} A^m$$

exists for each of the following matrices A, and compute the limit if it exists.

(6%) (a)

$$A = \begin{pmatrix} -2 & -1 \\ 4 & 3 \end{pmatrix}$$

(6%) (b)

$$A = \begin{pmatrix} 0.1 & 0.7 \\ 0.7 & 0.1 \end{pmatrix}$$

10. (6%) The matrix A_t is given:

$$\begin{pmatrix} -1/3 & t \\ -2 & 1 \end{pmatrix},$$

for which $t \in R$ there do not exist any *real* eigenvalues for A_t?

11. (6%) Solve the system of equations:

$$\begin{cases} 3x_2 + 3x_3 + 2x_4 = 219 \\ 2x_1 + 2x_2 + 4x_3 + 4x_4 = 326 \\ 8x_1 + 4x_2 + 4x_3 = 372 \\ x_1 + 2x_2 + 4x_3 + 3x_4 = 276 \end{cases}$$

12. (6%) Let A be $n \times n$ matrix with characteristic polynomial

$$f(t) = (-1)^n t^n + a_{n-1}t^{n-1} + \cdots + a_1 t + a_0.$$

Find the trace of A.

13. (6%) Let W_1 and W_2 be the following subspaces of $\mathbf{R}^4$.
W_1=span$\{[1,2,3,6]^T,[4,-1,3,6]^T,[5,1,6,12]^T\}$, W_2=span$\{[1,-1,1,1]^T,[2,-1,4,5]^T\}$. Find a basis of $W_1 \cap W_2$.