請從答案卷第一頁開始作答。不要計算過程。

一. 選擇題，共 7 題 (每題五分，共三十五分)

1. $\lim_{x\to 0}\left(x-\frac{1}{x}\right)\sin x = ?$ (A) -1 (B) 0 (C) 1 (D) not exist.

2. How many zeros does the function $f(x)=2^x-x^2-1$ have on the real line?
   (A) 2 (B) 3 (C) 4 (D) 5.

3. Evaluate $\int_0^2 (x-1)^{-2}\,dx$. (A) 0.5 (B) 1 (C) 2 (D) diverges.

4. If $f(x)$ is continuous and $\int_0^9 f(x)dx = 4$, then $\int_0^3 x f(x^2)\,dx = ?$

   (A) 2 (B) 4 (C) 8 (D) none of the above.

5. 求 $y=x^2$ 與 $y=x+1$ 所圍成之面積為

   (A) $\frac{5}{6\sqrt{5}}$ (B) $\frac{6}{5\sqrt{5}}$ (C) $\frac{6\sqrt{5}}{5}$ (D) $\frac{5\sqrt{5}}{6}$

6. Find the values of x such that the series $\sum_{n=1}^{\infty}(-1)^n\frac{1}{3^n\sqrt{n}}(x-1)^n$ converges.

   (A) $-3<x\le 3$ (B) $-3\le x\le 3$ (C) $-2<x\le 4$ (D) $-2\le x\le 4$.

7. Let $A=\begin{pmatrix} a & 2 & 1\\ 2 & a & 1\\ 1 & 1 & 2\end{pmatrix}$, where a is a real number. For which values of a, the matrix A is positive

   definite. (A) $a>2$ (B) $a<-2$ (C) $a<-1$ or $a>2$ (D) $a>0$.

二. 填充題，共 11 題 (第 1-10 題每題六分, 第 11 題五分, 共六十五分)

1. Let $\{a_n\}$ be a sequence of real numbers satisfying $a_{n+1}=\sqrt{a_n+6}$. If the initial value is $a_1=-2$,

   then $\lim_{n\to\infty} a_n =$ ________.

2. Let $f(x)=\begin{cases} c\,x\,e^{-4x^2}, & \text{if} \quad x\geq 0, \\ 0, & \text{otherwise,} \end{cases}$

where c is some constant such that $\int_{-\infty}^{\infty} f(x)\,dx=1$. Find $c=$ ________.

3. Find the maximum of xy under the condition that $x^2+2y^2=1$. The maximum is ________.

4. Find the solution of the differential equation: $\dfrac{f'(x)}{1-f(x)}=e^{2x}$, $f(0)=0$. $f(x)=$ ________.

5. Let $A=\begin{pmatrix} 2 & 6 \\ a & b \end{pmatrix}$. Find a and b such that A has eigenvectors $\boldsymbol{x}_1=(3,1)'$ and $\boldsymbol{x}_2=(2,1)'$.

$a=$ ________, $b=$ ________.

6. Following the previous problem (5), assume B is a different matrix with these same eigenvectors $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ but with eigenvalues $\lambda_1=1$ and $\lambda_2=0$. Find $B^{10}-2B^2=$ ________.

7. Assume U is the space spanned by $\{(1,-2,0,3),(0,1,0,-1)\}$ and V is the space spanned by $\{(1,0,0,0),(0,1,0,0),(0,0,1,0)\}$. Find the basis for the space of the intersection of U and V. The basis is ________.

8. Assume that $A=\begin{pmatrix} -1 & 0 \\ -1 & 1 \\ 1 & -2 \end{pmatrix}$ and $\boldsymbol{y}=\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$. Find the least squares solution for $A\boldsymbol{x}=\boldsymbol{y}$. $\boldsymbol{x}=$ ________.

9. 對任意 $a>1$，$\lim_{k\to 0}\left\{k\,(a^{1/k}-1)\right\}=$ ________.

10. 對任意 $a>0$，$\lim_{k\to\infty}\left\{k\,(a^{1/k}-1)\right\}=$ ________.

11. 如何用數值方法求 ln(3)? (答案請少於 50 字)