國　立　清　華　大　學　命　題　紙

96學年度＿統計學研究所＿碩士班入學考試

科目＿機率論＿科目代碼＿0102＿共＿1＿頁第＿1＿頁 *請在【答案卷】內作答

1. Write down the probability functions, or the probability density functions, means and variances of the following distributions:
   (i)Poisson, (ii)multinomial and (iii) gamma. (30%)

2. Suppose that X is a random and g is a nonnegative function such that $E(g(X))$ exists.
   (i)Show that $P(g(X)\geq l)\leq E(g(X))/l$.
   (ii)Use (i) to prove the Chebyshev's inequality.
   (iii)If the distribution of X is symmetric about 0 and $E(X^2)=\sigma^2$.
   Use (i) to show that, for all $x>0$,
   $$P(X\leq x)\geq\frac{x^2}{x^2+\sigma^2}.$$ (15%)

3. State and prove the Central Limit Theorem and, use it, to show that
   $$\lim_{n\to\infty}e^{-n}\sum_{k=0}^{n}\frac{n^k}{k!}=\frac{1}{2}.$$ (15%)

4. Let $X_1,X_2,\ldots$ be a sequence of iid $U(0,1)$ variables and $N=\min\{n\geq 2;X_n>X_{n-1}\}$.
   Find the distribution and the expected value of N.
   (10%)

5. Let $X_1,X_2,\ldots,X_n$ be iid $N(\mu,\sigma^2)$ variables and
   $$\overline{X}=\frac{1}{n}\sum_{i=1}^{n}X_i,\ S^2=\frac{1}{n-1}\sum_{i=1}^{n}(X_i-\overline{X})^2.$$
   Find the distributions of $\overline{X}$ and S^2, and show that they are statistically independent.
   (20%)

6. At a party, n men throw their hats into the center of a room. The hats are mixed up and each man randomly selects one. Find
   (i)the probability that none of the n men selects his own hat.
   (ii)the expected number of men who select their own hats.
   (10%)