1. Let X be a standard normal random variable and let I, independent of X, be such that $P\{I=1\}=1/2=P\{I=-1\}$. Now define random variable Y by Y=I·X.

   (a) What is the distribution of Y? (5%)
   (b) Compute Cov( X, Y). (5%)
   (c) Are X and Y independent? Explain. (5%)
   (d) Are I and Y independent? Explain. (5%)

2. Let $X_1, X_2, \ldots$ be a sequence of i.i.d. random variables with mean $\mu$ and variance $\sigma^2$ and let N, independent of $X's$, be a binomial random variable with parameters (n, p). The random sum, denoted $S_N$, is given by $S_N = \sum_{i=1}^{N} X_i$, $(S_0 \equiv 0)$. Find

   (a) The expectation $ES_N$. (10%)

   (b) The variance. $Var(S_N)$ (10%)

3. A miner is trapped in a mine containing 3 doors. Door 1 leads to a tunnel that will take him to safety after 1 hour of travel. Door 2 leads to a tunnel that will return him to the mine after 2 hours. Door 3 leads to a tunnel that will return him to the mine after 3 hours. We assume that the miner is at all times equally likely to choose any one of the doors. Let T be the length of time until he reaches safety, find the moment generating function of T. (10%)

4. Suppose that the moment generating function of a random variable $X$ is given by $M(t)=\dfrac{2e^t}{4-e^{2t}-e^{3t}}$. Find the probabilities $P\{X=i\}$, $i=1,2,3,4,5$. (10%)

5. Let $X_1, X_2, \ldots$ be a sequence of i.i.d. random variables having uniform distribution over interval $(0,a)$. Define $M_n = n \cdot \min\{X_1, X_2, \ldots, X_n\}$. What is the limiting distribution, as $n \to \infty$, of $M_n$? (10%)

6. Suppose that there are k different coupons and each time one obtains a coupon it is equally likely to be any one of the k types. Let X be the number of different types of coupons that are contained in a set of n coupons. Compute
   (a) the expectation EX. (10%)
   (b) the variance Var(X). (10%)

7. Consider a particle initially located at a given point in the plane and suppose that it undergoes a sequence of steps of length 1 but in a completely random direction (equally likely in any direction). Calculate the expected square of the distance from the origin after n steps. (10%)