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九十三學年度　統　計　（所）　　組碩士班入學考試

科目　基礎數學　科號　0301　共　3　頁第　1　頁　*請在試卷【答案卷】內作答

※ 作答時，請將答案統一寫在答案卷的第一頁，並清楚標示題型及題號。請使用第二頁以後之試卷來解題，解題的計算過程不列入評分。

一、 選擇題（每題六分，共三十六分）

1. Consider the vector space $S=\{(a,a+b,a+b,-b)', -\infty<a<\infty, -\infty<b<\infty\}$. Determine which of the following sets of vectors are spanning sets of S.
   (a) $\{(1,0,0,1)',(1,2,2,-1)'\}$.
   (b) $\{(1,1,0,0)',(0,0,1,-1)'\}$.
   (c) $\{(2,1,1,1)',(3,1,1,2)',(3,2,2,1)'\}$.
   (d) $\{(1,0,0,0)',(0,1,1,0)',(0,0,0,1)'\}$.

2. Which of the following is the nearest to $\int_0^{0.1}\frac{\ln(1+t)}{t}dt$?
   (a) $\frac{0.1}{1}-\frac{(0.1)^2}{2}+\frac{(0.1)^3}{3}-\frac{(0.1)^4}{4}$.
   (b) $\frac{0.1}{1}+\frac{(0.1)^2}{2}+\frac{(0.1)^3}{3}+\frac{(0.1)^4}{4}$.
   (c) $\frac{0.1}{1^2}-\frac{(0.1)^2}{2^2}+\frac{(0.1)^3}{3^2}-\frac{(0.1)^4}{4^2}$.
   (d) $\frac{0.1}{1^2}+\frac{(0.1)^2}{2^2}+\frac{(0.1)^3}{3^2}+\frac{(0.1)^4}{4^2}$.

3. Let $f(x)=(x-1)\cdot\sqrt[3]{x^2}$. Which of the following is not correct?
   (a) f has a relative maximum at $x=0$.
   (b) f has a relative minimum at $x=\frac{2}{5}$.
   (c) f has an inflection point at $x=\frac{1}{5}$.
   (d) $f(x)\to-\infty$ *as* $x\to-\infty$.

4. Let f be a continuous function on [0, 3] and $\int_0^3 f(x)\,dx=9$. Which of the following numbers can always be attained by f in the interval [0, 3] ?

(a) 1.

(b) 3.

(c) 5.

(d) 7.

5. Which of the following integrals can NOT be evaluated?

(a) $\int e^{\sqrt{x}} dx$

(b) $\int \sin^8 x \, dx$

(c) $\int \frac{x^{11} + x + 1}{x^9 + x^8 + x^5} dx$

(d) $\int \frac{\sin x}{x} dx$.

6. Which of the following statement is NOT correct?

(a) Assume $\{\lambda_i : i = 1,2,\cdots,n\}$ are the eigenvalues of an $n \times n$ matrix A.

Then $\{\lambda_i + \alpha : i = 1,2,\cdots,n\}$ are the eigenvalues of $A + \alpha I$ where I is the $n \times n$ identity matrix.

(b) Assume A and B are $n \times n$ covariance matrices (i.e., positive definite and symmetric). Then, the sum of the eigenvalues of $A + B$ is larger than the sum of the eigenvalues of A.

(c) Assume A and B are $n \times n$ matrices. Then, the eigenvalues of AB are identical to the eigenvalues of BA.

(d) 以上皆非

二、 填充題（每題八分，共六十四分）

1. Let S be the vector space spanned by vectors $(1,2,3)'$ and $(1,1,-1)'$. Find the point ___(1)___ in S that is closest to the point $(1,1,1)'$.

2. Assume that $X=(X_1,X_2,\cdots,X_n)'$, $\bar{X}_n=\frac{1}{n}\sum_{i=1}^{n}X_i$. If $X-\bar{X}_n\equiv AX$, find the rank of A. $rank(A)=$ (2)

3. Find the eigenvalues of the $m\times m$ matrix A, where

$$A=\begin{pmatrix}1 & \rho & \cdots & \rho\\ \rho & 1 & \cdots & \rho\\ \vdots & \vdots & \ddots & \vdots\\ \rho & \rho & \cdots & 1\end{pmatrix}.$$ Eigenvalues of $A=$ (3)

4. Find the maximum of $f(x,y,z)=x+2y+3z$ on the surface $x^2+y^2+z^2=1$.

The maximum is (4)

5. Compute $\int_1^2 x^n\ln(x)\,dx=$ (5)

6. Compute $\int_{-\infty}^{\infty}(\cos x)\,e^{-x^2/2}dx=$ (6)

7. $\sum_{n=1}^{\infty}(n-1)\left(\frac{1}{2}\right)^n=$ (7)

8. Assume that $A=\begin{pmatrix}1/2 & 1/4\\ 1/4 & 1/2\end{pmatrix}$. Compute $A^{100}=$ (8)