

國立清華大學 命題紙

九十二學年度 統計學研究所 系(所) _____ 組碩士班研究生招生考試

科目 基礎數學 科號 0301 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

(1) Evaluate:

(6%) (a) $\int_{-\infty}^{\infty} x^2 e^{-x^2} dx$

(6%) (b) $\int_0^1 \frac{1}{\sqrt{x(1-x)}} dx$

(6%) (c) $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right) \left(1 + \frac{2}{n} \right) \dots \left(1 + \frac{\lfloor \sqrt{n} \rfloor}{n} \right)$

(6%) (d) $\lim_{n \rightarrow \infty} \left(\frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{3n} \right)$

(6%) (e) $\lim_{n \rightarrow \infty} x_n$, where $x_1 = \sqrt{6}$, $x_{n+1} = \sqrt{6 + x_n}$, $n = 1, 2, \dots$

(10%) (2) Compute the line integral

$$\int_C \left(\frac{-y}{x^2 + y^2} \right) dx + \left(\frac{x}{x^2 + y^2} \right) dy,$$

where C is the curve $\frac{x^2}{16} + \frac{y^2}{25} = 1$ traversed counterclockwise.

(10%) (3) Find a function $f(x)$ such that

$$\frac{f'(x)}{1-f(x)} = 1+2x, f(0) = 0.$$

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(10%) (4) Let W_1 be the set of 3×3 matrices of the form $\begin{bmatrix} x & 0 & z \\ 0 & -x & 0 \\ y & 0 & x \end{bmatrix}$, $x, y, z \in R^1$,

W_2 be the set of 3×3 matrices of the form $\begin{bmatrix} -a & 0 & b \\ 0 & a & 0 \\ c & 0 & a \end{bmatrix}$, $a, b, c \in R^1$.

Show that W_1 and W_2 are vector spaces over R^1 and find the dimensions of W_1 , W_2 , $W_1 + W_2$ and $W_1 \cap W_2$.

(10%) (5) Let A be any $m \times n$ matrix over R^1 . Show that the column rank of A is equal to the row rank of A . (the column/row rank of A is the dimension of the column/row space)

(10%) (6) Find a polynomial $p(x)$ with real coefficients such that $p(x)$ has degree ≤ 4 and $p(-2) = 24$, $p(-1) = -6$, $p(0) = 4$, $p(1) = -6$, $p(2) = 24$.

(10%) (7) Evaluate $\lim_{n \rightarrow \infty} A^n x$, where $A = \begin{bmatrix} 1/3 & 2/3 \\ 2/3 & 1/3 \end{bmatrix}$, $x = (3, 7)^t$.

(10%) (8) Let $A = \begin{bmatrix} 2 & -2 & 1 \\ 0 & 1 & 0 \\ -1 & 2 & 0 \end{bmatrix}$, compute A^{100} .