八十六學年度統計學研究所碩士般研究生入學考試

科目 　基礎數學　 科號 0301共2頁第1頁*請在試卷（答案卷）內作答

1.(10%) Sketch the graph of $f(x) = \dfrac{x^2 - 1}{x^3}$.

2.(10%) Evaluate the series

$$\sum_{k=0}^{\infty} \frac{x^k}{k+1}, \quad 0 < x < 1.$$

3.(10%) Evaluate

$$\int_0^1 \sqrt{4x - x^2}\, dx$$

4.(10%) Evaluate the line integral

$$\int_C y \tan^2 x\, dx + \tan x\, dy,$$

where C is the circle $x^2 + y^2 = 1$ traversed counterclockwise.

5.(10%) Find the maximum and minimum values of

$$f(x, y, z) = 3x + 6y + 2z$$

subject to the constraint $2x^2 + 4y^2 + z^2 = 70$.

6. Let P be an $n \times n$ idempotent matrix, i.e. $P^2 = P$. Prove

6% (a) The eigenvalues of P are either 0 or 1.

6% (b) If rank $(P) = n$ then $P = I_n$, the $n \times n$ indentity matrix.

6% (c) If P is symmetric then $trace(P) = rank(P)$.

7. Let $(\sqrt{2}, \sqrt{2}, 0, 0)'$ and $(\sqrt{2}, 0, \sqrt{2}, 0)'$ be a basis of V_2, which is a subspace of

   V_4 (4-dimensional Euclidean space). Suppose that $\underset{\sim}{x} = (1,1,1,1)'$ is a vector in

   V_4.

   6% (a) Find the projection matrix T of V_4 onto V_2.

   6% (b) Find the projection of $\underset{\sim}{x}$ on V_2.

   6% (c) Let $V_2^\perp$ be the orthogonal complement of V_2 in V_4. Find the

   projection of $\underset{\sim}{x}$ on $V_2^\perp$.

8. Let $A = (a_{ij})$ be a $n \times n$ matrix with $a_{ij} = 1$, $1 \le i, j \le n$.

   7% (a) Show that A satisfies the equation $x^{k-1}(x - n) = 0$, $k = 1, 2, \ldots$.
   7% (b) Find the eigenvalues of A.