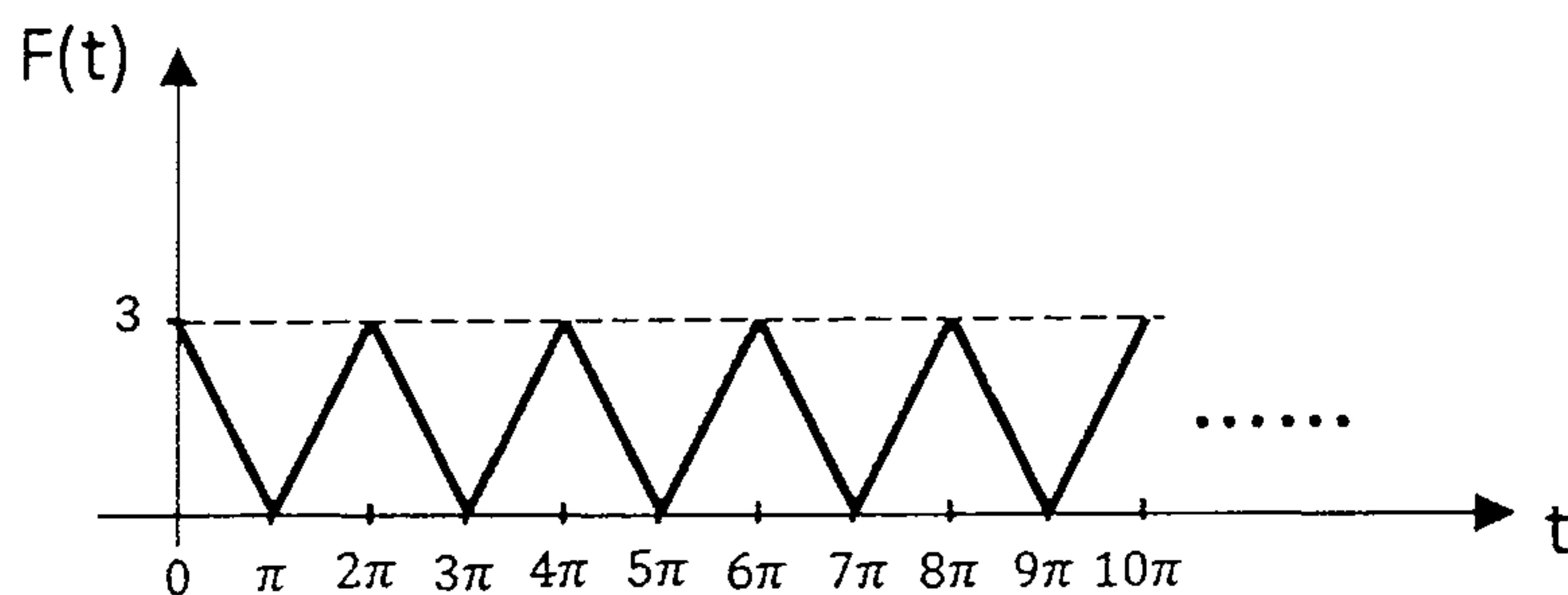


※請在答案卷內作答

1 到 8 題皆為計算題，請詳列計算過程，無計算過程者不給分。

1. (10 %) Evaluate the integral $\oint_C [(5x^3 - 2y)dx - (5x + 2y^3)dy]$,
where $C : x^2 + y^2 = 25$, $z = 0$.

2. (15 %) Find the Laplace transform of the periodic function $F(t)$ as
shown in the following figure.



3. For a matrix $A = \begin{bmatrix} -7 & -2 & 3 \\ 12 & 7 & -3 \\ -10 & -4 & 4 \end{bmatrix}$

(a) (10 %) Diagonalize the matrix A to be a diagonal matrix

$D = S^{-1}AS$. Find the matrix D and S .

(b) (5 %) Find the matrix A^6 .

4. Consider a complex valued function $f(z) = \frac{2}{z^2 - 2}$.

(a) (7 %) Expand the function in the Taylor series centered at $z = 0$.

(b) (3 %) Find the region of convergence for this series.

注意：背面有試題

參考用

※請在答案卷內作答

5. (10 %) Solve the boundary value problem:

$$y'' + 9y = 1 \text{ for } 0 \leq x \leq L, \quad y(0) = 1 \text{ and } y(L) = 0.$$

6. (15 %) Solve the initial value problem:

$$x^2 y'' - xy' - 3y = \frac{1}{x} \text{ for } x > 0, \quad y(1) = \frac{1}{12} \text{ and } y'(1) = 1.$$

7. Denote the Fourier transform of $f(t)$ as

$$\mathcal{F}\{f(t)\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt = F(\omega).$$

(a) (5 %) Determine the Fourier transform of

$$f_1(t) = \cos(3t) \exp(-2|t|).$$

(b) (5 %) Find and sketch the Fourier transform of

$$f_2(t) = \sqrt{2\pi} [\delta(t+2) + \delta(t-2)].$$

8. (15 %) A thin metal bar of length L is homogeneous, laterally insulated, and has the thermal diffusivity $c^2 = K/(\sigma\rho) = 100 \text{ mm}^2 / \text{sec}$. The bar has both ends $x=0$ and $x=L$ kept at zero temperature all the time.

The temperature distribution at time $t=t_0$ is $u(x, t_0) = 0.1 \sin\left(\frac{\pi x}{L}\right)$.

Find $u(x, t)$ for $0 \leq x \leq L$ and $t \geq t_0$ by letting $u(x, t) = X(x) T(t)$

and solving the heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$. Show details.

注意：背面有試題

參考用