

※請在答案卷內作答

1. A set of three vectors in Cartesian coordinates are

$$\vec{u}_1 = (1, 1, -1), \vec{u}_2 = (-1, 1, 1), \vec{u}_3 = (1, -1, 1). \text{ Find a set of three}$$

vectors \vec{v}_1, \vec{v}_2 and \vec{v}_3 , such that their inner products

$$\text{satisfying } \vec{u}_i \cdot \vec{v}_j = \delta_{i,j}. \quad (10\%)$$

2. (a) Expand $\exp[\tan x]$ up to $o(x^4)$ for $|x| < 1$. (5%)

(b) Expand $\tan^{-1} x$ up to $o(x^7)$ for $|x| < 1$. (5%)

3. (a) Find $\frac{dI}{dx}$ where $I = \int_{x^2}^{\sin^{-1} x} \frac{\sin t}{t} dt$. (5%)

- (b) Find the equation of tangent line to the curve

$$x^3 - 3y^3 + xy + 21 = 0 \text{ at } (x, y) = (1, 1). \quad (5\%)$$

4. A constant density solid ellipsoid inside the surface of $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{25} = 1$, calculate the moment of inertia in terms of total mass for the rotation about the z-axis. (10%)

5. Find the solution of the ordinary differential equation

$$\frac{d^2 y}{dx^2} + \frac{dy}{dx} - 2y = 4 \sin(2x). \quad (10\%)$$

注意：背面有試題

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6. Let us consider a functional defined by the integral,

$$I[x] = \int_{t_0}^{t_1} L(x(t), \dot{x}(t), t) dt,$$

where the values $t_0, t_1, x(t_0), x(t_1)$ are given. Here dot means the derivative with respect to t . Prove that the function $x(t)$ that minimizing/maximizing the functional $I[x]$ should satisfy

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) - \frac{\partial L}{\partial x} = 0.$$

(a) From the condition $\delta I = 0$, obtain the above equation. (5%)

(b) If the functional is defined by

$$I[x] = \int_{t_0}^{t_1} L(x(t), \dot{x}(t), \ddot{x}(t), t) dt,$$

where the values $t_0, t_1, x(t_0), x(t_1), \dot{x}(t_0)$, and $\dot{x}(t_1)$ are given. Obtain the differential equation that is satisfied by the function maximizing/minimizing the functional (10%).

7. Hermite polynomial is defined by the Rodrigues' formula,

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}.$$

(a) Using the Goursat's theorem,

$$\frac{d^n}{dx^n} f(x) = \frac{n!}{2\pi i} \int_C \frac{f(z)}{(z-x)^{n+1}} dz,$$

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obtain the generating function $J(x, s)$ defined by

$$J(x, s) = \sum_{n=0}^{\infty} \frac{s^n}{n!} H_n(x). \quad (5\%)$$

(b) Evaluate the integral, $\int_{-\infty}^{\infty} H_n(x) H_m(x) e^{-x^2} dx$. (10%)

8. Obtain the Fourier transform of the following functions: (5% each)

$$(a) f(x) = \begin{cases} 1 & 0 \leq |x| < 1 \\ 0 & 1 \leq |x| \end{cases} \quad (b) f(x) = \begin{cases} x & 0 \leq |x| < 1 \\ 0 & 1 \leq |x| \end{cases}$$

$$(c) f(x) = \begin{cases} x^2 & 0 \leq |x| < 1 \\ 0 & 1 \leq |x| \end{cases} \quad (d) f(x) = e^{-x^2}$$