

※選擇題請在答案卡內作答，非選擇題請在答案卷內作答

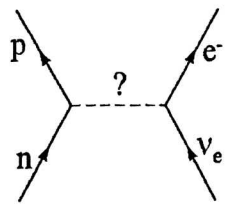
Part A: 單選題，答案請填於答案卡。一題 5 分，答錯倒扣 1 分，不作答不計分亦不扣分。

Planck constant $h = 6.63 \times 10^{-34}$ J·s; speed of light $c = 3 \times 10^8$ m/s; electron charge $e = -1.60 \times 10^{-19}$ C; electron mass $m_e = 9.11 \times 10^{-31}$ kg = 511 keV/c²; Bohr radius $a_0 = 5.29 \times 10^{-11}$ m;

參考用

1. The discovery that electron possesses spin is important for the explanation of which of the following topics? (A) The cyclotron motion of a moving electron in a magnetic field (B) The photoelectric effect (C) The existence of isotopes (D) The structure of periodic table (E) The existence of anti-particle

2. In the process of beta decay, a neutron decays into a proton and other light particles as shown in the right figure. What is the mediator for this interaction? (A) π^- (B) W^- (C) μ^- (D) photon (E) gluon



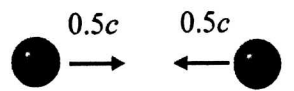
3. Suppose 149 g sample of ¹⁴⁹Sm (atomic mass = 149 u) is observed to decay at a rate of 33 Bq, what is the half-life of the sample in seconds? (1 Bq = 1 decay count/sec, 1 mole = 6 × 10²³) (A) 1.26 × 10²² (B) 3.03 × 10²³ (C) 1.82 × 10²² (D) 5.5 × 10⁻²³ (E) 3.03 × 10⁻²

4. Taiwan High Speed Rail travels from Taipei to Kaohsiung (distance = 300 km) at a speed of 216 km/hour. What is the difference in travel time as measured by an observer on the ground and a passenger on the train? (A) 5 ns (B) 0.1 ns (C) 0.3 ns (D) 2 ns (E) 20 ns

5. An atom at rest is originally in the excited state and then emits a photon spontaneously and returns to the ground state. The energy difference between the two states is ΔE . Including the recoil effect, what is the frequency of the emitted photon? The atom has mass m . (A) $\frac{\Delta E}{h}$ (B) $\frac{\Delta E + mc^2}{h}$ (C)

$$\frac{\Delta E}{h} - \frac{(\Delta E)^2}{2hmc^2} \text{ (D) } \frac{\Delta E}{h} - \frac{m^2c^4}{2h(\Delta E)} \text{ (E) } \frac{\Delta E - mc^2}{h}$$

6. Two protons are moving toward each other, each with speed $0.5c$ relative to the laboratory frame. If an observer is moving at $0.5c$ together with one of the proton, what is the kinetic energy of the other proton as measured by the observer? The rest energy of one proton is $m_p c^2 = 938$ MeV. (A) 145 MeV (B) 290 MeV (C) 375 MeV (D) 625 MeV (E) 750 MeV



7. How many photons are emitted per second from a green laser pointer with power of 1 mW and wavelength of 532 nm? (A) 2.67 × 10¹⁵ (B) 2.67 × 10¹⁸ (C) 3.53 × 10²² (D) 8.02 × 10²³ (E) 1.77 × 10²⁷

8. The first excited state of helium has the configuration (1s)¹(2s)¹, in which the two electron spins are parallel. Which of the following statement is correct? (A) Total spin is 1/2. (B) This state has no Zeeman effect. (C) The term symbol is 2³S₁. (D) Total angular momentum is zero. (E) Total magnetic moment is zero.

9. A photon of energy 2.4×10^3 eV collides with a free electron. What is the maximum energy the electron can acquire in this collision? (A) 9 eV (B) 11 eV (C) 18 eV (D) 21 eV (E) 23 eV

注意：背面有試題

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Part B: 計算題，請詳列計算過程，無計算過程不予計分。

1. (20%) de Broglie suggested that $E = h\nu$ and $p = h/\lambda$ for both photons and massive particles. As a result, the electron also has wave-like behavior.

(a) (10%) For an electron with mass m_e and momentum p , find the phase velocity and group velocity of the electron. You need to use relativistic energy.

(b) (5%) Verify that the group velocity cannot exceed c .

(c) (5%) Show that for classical limit $p \ll m_e c$, the group velocity reduces to p/m_e .

2. (20%) The first three low-lying electron's wave functions of hydrogen atom are given as:

$$\psi_{1s} = \frac{1}{\sqrt{\pi}} (a_0)^{-3/2} \cdot e^{-r/a_0}, \quad \psi_{2s} = \frac{1}{2\sqrt{2\pi}} (a_0)^{-3/2} \cdot e^{-r/2a_0} \left(1 - \frac{r}{2a_0} \right), \quad \psi_{2p} = \frac{1}{4\sqrt{2\pi}} (a_0)^{-3/2} \cdot \frac{r}{a_0} \cdot e^{-r/2a_0} \cdot \cos\theta$$

with the normalization condition:
$$\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{r=0}^{\infty} \psi^* \psi(r) r^2 \sin\theta dr d\theta d\phi = 1.$$

(a) What is the most likely location from the origin for the 1s electron? (in terms of a_0)

(b) What is the mean radius of the 1s electron? (in terms of a_0) You may need this integral:

$$\int_0^{\infty} x^3 e^{-x} dx = 3! = 6$$

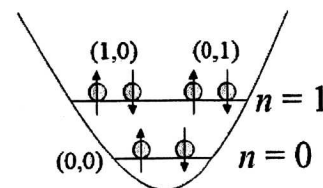
(c) Find the charge density of the electron at the origin ($r = 0$) in unit of coulomb/m³ for the three states, 1s, 2s, and 2p.

(d) For an electric-dipole allowed transition between state 1 and state 2, the transition probability is proportional to the transition dipole matrix element: $\int \psi_1^* (r \cdot |E_0| \cos\theta) \psi_2 d^3\vec{r}$, where E_0 is the electric field of the applied laser light. Show that 1s→2s transition is forbidden while 1s→2p is allowed.

3. (15%) Consider N identical spin-1/2 fermions, each of mass m . Suppose they are trapped in a two-dimensional simple harmonic potential, $V(x, y) = \frac{1}{2} m \omega^2 (x^2 + y^2)$. The quantized energy levels are:

$$E_n = (n_x + n_y + 1) \hbar \omega, \text{ where } n_x, n_y = 0, 1, 2, 3 \dots$$

For $n = n_x + n_y = 1$, there are four degenerate states with equal energy $E_1 = 2\hbar\omega$. These are $(n_x, n_y) = (1, 0)$ and $(0, 1)$ and they can be in the spin-up or spin down state as shown in the figure.



(a) (4%) Write down the general expression for the number of degenerate states with energy $E_n = (n+1)\hbar\omega$.

(b) (7%) Sum the total number of possible states with energy $E \leq E_n$ and set it equal to N . The highest energy E_n is the Fermi energy of the system. Derive the Fermi energy when $N \gg 1$.

(c) (4%) At what temperature will the N particles become degenerate Fermi gas as calculated in (b)?