1. (a)V is a scalar field. Prove that $\nabla \times (\nabla V) = 0$. (5%)

   (b)**A** is a vector field. Prove that $\nabla \cdot (\nabla \times \mathbf{A}) = 0$.(5%)

2. A long coaxial cable carries a uniform volume charge density ρ on the inner cylinder (radius a), and a uniform surface charge density on the outer cylindrical shell (radius b). This surface charge is negative and of just the right magnitude so that the cable as a whole is electrically neutral. Find the electric field in each of the three regions (i) inside the inner cylinder (3%), (ii) between the cylinders (3%), (iii) outside the cable (3%)

3. Two infinite parallel grounded conducting planes are held a distance a apart. A point charge q is placed in between of them, with a distance x to one plate. (a)Find the force on q. (5%) (b) Find the force as $a \to \infty$, (3%)(c) Find the force as $x = a/2$. (3%)

4. A capacitor consists of two coaxial metallic cylindrical surfaces of a length 50 (mm) and radii 3 (mm) and 8 (mm). The dielectric material between the surfaces has a relative permittivity $\epsilon_r = 2000 + \left(\frac{4000}{r}\right)$, where r is the measured distance from the axis in meter. Determine the capacitance of the capacitor. (10%)

5. A spherical shell, of radius R, carrying a uniform surface charge σ, is set spinning at angular velocity ω. Find the magnetic dipole moment of this spinning spherical shell. (10%)

6. A uniform plane electromagnetic wave propagates in the +z direction and is incident normally on a surface of dielectric medium at z=0. Let the magnetic field at z=0 be $\mathbf{H}(0,t) = \mathbf{a_y} H_0 \cos 10^4 t$ (A/m). (a) Determine the skin depth (For the medium: conductivity=σ, permeability=μ_0.) (5%)(b) Find the expression for $\mathbf{H}(z,t)$, and $\mathbf{E}(z,t)$. (5%) (c) Find the power loss per unit area (in terms of H_0) in the medium. (8%)

7. The SWR on a lossless 50 (Ω) transmission line terminated in an unknown load impedance is found to be 3.0. The distance between successive voltage minima is 20 (cm), and the first minimum is located at 5 (cm) from the load. Determine (a) the reflection coefficient Γ (5%), and (b) the load impedance Z_L. (5%) (c) Find the equivalent length and terminating resistance of a line such that the input impedance is equal to Z_L.(7%)

8. Consider a rectangular wave guide with dimensions 2.28 cm x 1.01 cm. What TE modes will propagate in this wave guide, if the driving frequency is 1.70 x 10^{10} Hz? (5%) Suppose you want to excite only one TE mode; what range of frequencies could you use? (5%) What are the corresponding wavelengths (in open space)? (5%)