

國 立 清 華 大 學 命 題 紙

97 學年度 生醫工程與環境科學系 (所) 丙(醫學物理與工程)組碩士班入學考試

科目 應用數學 科目代碼 2702 共 2 頁第 1 頁 *請在【答案卷卡】內作答

1. (10 %) Write the general solutions to

$$\mathbf{Ax} = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

as the sum of a particular solution to $\mathbf{Ax}=\mathbf{b}$ and the general solution to $\mathbf{Ax}=\mathbf{0}$.

2. (10%) Find the general solution of the following differential equation

$$y'' + xy' - y = e^{3x}. \text{ Hint } e^{3x} = \sum_{k=0}^{\infty} \frac{3^k}{k!} x^k.$$

3. (10 %) Given $P_n(x) = \sum_{m=0}^M (-1)^m \frac{(2n-2m)!}{2^n m!(n-m)!(n-2m)!} x^{n-2m}$, when n is even,

$m=n/2$, else $m=(n-1)/2$ show that:

(a) $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} [(x^2 - 1)^n]$

(b) $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$

4. (10%) (a) Compute the eigenvectors and eigenvalues of \mathbf{A} .

(b) Is it possible to write \mathbf{A} in the form \mathbf{PDP}^{-1} , where \mathbf{D} is diagonal and \mathbf{P} is invertible? If yes, what are \mathbf{D} and \mathbf{P} ?

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

5. (10%) Consider the Gamma function $\Gamma(\alpha) = \int_0^{\infty} e^{-\tau} \tau^{\alpha-1} d\tau$, $\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$

and the fact $\Gamma(1/2) = \sqrt{\pi}$. Find the Laplace transforms (a) $L\{t^\nu\}$ and

(b) $L\{t^{-1/2}\}$.

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6. (10 %) A mass attached to a spring is released from rest 1m below the equilibrium position ($y(0)=1, y'(0)=0$) for the mass-spring system and begins to vibrate. After $\pi/2$ seconds, the mass is struck by a hammer exerting an impulse on the mass. The system is governed by the initial value problem

$$y'' + 9y = -3\delta\left(t - \frac{\pi}{2}\right), y(0)=1, y'(0)=0$$

where $y(t)$ denotes the displacement from equilibrium at time t . Solve $y(t)$ and observe what happens to the mass after it is struck.

7. (10 %) Solve the initial value problem $\frac{dy}{dt} = 1 + y + t^2y + t^2, y(0)=0$.

8. (10 %) Find the general solution to the following differential equation

$$x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = x^{5/2}, \quad x > 0$$

9. (10 %) (a) Solve the initial value problem

$$y'' + \omega^2y = \sin \gamma, \quad y(0)=0, y'(0)=0, \omega \neq \gamma$$

(b) If $\omega \rightarrow \gamma$, what will be the solution?

10. (10 %) Given $J_{\nu-1}(x) + J_{\nu+1}(x) = \frac{2\nu}{x}J_{\nu}(x)$ and $J_{\nu-1}(x) - J_{\nu+1}(x) = 2J'_{\nu}(x)$, show

that

$$\frac{d}{dx}[xJ_{\nu}(x)J_{\nu+1}(x)] = x[J_{\nu}^2(x) - J_{\nu+1}^2(x)]$$