國　立　清　華　大　學　命　題　紙

96 學年度生醫工程與環境科學系甲組(分子生醫光電)組碩士班入學考試

科目應用數學 科目代碼 2503 共 1 頁第 1 頁 *請在【答案卷卡】內作答

1. (10%). Find the general solution of the following differential equation

   $x^2y'' + 6xy' + 4y = 0$.

2. (10%).Solve the initial value problem $x^2y'' - 3xy' + 3y = 2x^2 \ln x$ with $y(1) = 0$ and

   $y'(1) = 0$.

3. (10%). Solve the non-homogeneous system of differential equations

   $x_1'(t) + 2x_1 + 4x_2 = 2t - 1$, $x_2'(t) + x_1 - x_2 = \sin t$.

4. (10%). Find the eigenvalues and eigenvectors of the following matrix

   $$\mathbf{A} = \begin{bmatrix} -5 & 8 & 1 \\ -3 & 6 & 1 \\ 6 & -8 & 0 \end{bmatrix}.$$

5. (10%). Find the residues at the poles of the complex function (a) $f(z) = \dfrac{z^2 + 2z + 3}{z - i}$,

   and (b) $f(z) = \dfrac{1}{z \sin z}$.

6. (10%). Find the Fourier series representation of the function $f(x) = |x|$ in the interval

   $-L \le x \le L$.

7. (10%). Solve the following Volterra integral equation by using Laplace transform

   $y(t) = 2e^{-t} + \int_0^t \sin(t - \tau) y(\tau) d\tau$.

8. (10 %). Find the general solution $\dfrac{dy}{dx} - 2y = xy^{1/2}$.

9. (10%). Consider the differential equation $xy'' + 2y' + xy = 0$ show that

   $y_1(x) = \sin x / x$ is a solution and find a second linear independent solution.

10. (10%). Solve the initial value problem $y'' + 2y' + y = \sin x$ with $y(0) = 1$ and

    $y'(0) = 0$.