

1. (10 Points) Find the solution of the following differential equation.

$$(3x^2 - 2xy + 2)dx + (6y^2 - x^2 + 3)dy = 0$$

2. (10 Points) Find the general solution of

$$y'' - 3y' - 4y = -8e^x \cos 2x$$

3. (10 Points) Given that $y_1(x) = 1/x$ is a solution of

$$2x^2 y'' + 3xy' - y = 0$$

Find the second linear independent solution.

4. (10 Points) Find the particular solution of the following differential equation

$$x^2 y'' - 3xy' + 4y = x^2 \ln x, \quad x > 0, y_1 = x^2, y_2 = x^2 \ln x$$

5. (10 Points) Given $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$ and $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$, find $J_{3/2}(x), J_{-3/2}(x)$.

6. (10 Points) Find the Laplace transform of the following function

$$f(t) = \int_0^t (t-\tau)^2 \cos 2\tau d\tau.$$

7. (10 Points) Find the general solution of the following equation

$$\mathbf{y}' = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \mathbf{y} + \begin{bmatrix} 2e^{-x} \\ 3x \end{bmatrix}$$

8. (10 Points) Find eigenvalues and eigenvectors of the matrix

$$\mathbf{A} = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

9. (10 Points) Find the integral

$$\oint_C \frac{z^3 - 6}{2z - i} dz = ?$$

10. (10 Points) Find the solution of initial value problem

$$y'' + \omega^2 y = f(t), y(0) = 1, y'(0) = 0$$

where $f(t)$ is periodic with period 2π and

$$f(t) = \begin{cases} 1-t & 0 \leq t < 1 \\ -1+t & 1 \leq t < 2 \end{cases}$$