

國立清華大學命題紙

九十三學年度 原子科學 (系)(所) 甲、丙 組碩士班入學考試

科目 應用數學 科號 3503 共 1 頁第 1 頁 \*請在試卷【答案卷】內作答

1. (10%) Solve  $y(1 - xy + x^2y^2)dx + x(x^2y^2 - xy)dy = 0$ .
2. (10%) Solve  $y' + y \tan x = \sin 2x, y(0) = 1$ .
3. (10%) Solve  $y'' - 4y' + 4y = 0, y(0) = 3, y'(0) = 1$ .
4. (10%) Solve  $y'' + y = -2 \sin x + 4x \cos x$ .
5. (10%) Solve  $x^2y'' + xy' + (x-1)y = 0$
6. (10%) Assume that  $f(x) = f(x + 2\pi)$  is a periodic function. Find the Fourier series of  $f(x) = \begin{cases} \sin x & -\pi < x < 0 \\ \cos x & 0 < x < \pi \end{cases}$ .
7. (20%) Suppose that  $x$  and  $y$  are functions of  $t$ . Solve the system of differential equation  $\begin{cases} x' + y' + 3x = \cos t & x(0) = 0 \\ x' + y' = \sin t & y(0) = 4 \end{cases}$  using Laplace transform.
8. (20%) Find eigenvalues and eigenvectors of matrices  $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$  and  $\begin{bmatrix} 4 & -1 & -2 \\ 2 & 1 & -2 \\ 1 & -1 & 1 \end{bmatrix}$ .