

1. Solve the following ordinary differential equation. Please give all steps in details, and give your answer in the explicit form.

(a) $y' + 2xy = 2x/y$, $y(0) = 2$. (8 points)

(b) $y'' + 4y = r(t)$, $y(0) = 0$, $y'(0) = 3$. (10 points)

$$r(t) = \begin{cases} 3\sin t & 0 < t < \pi \\ -3\sin t & t > \pi \end{cases}$$

2. **Model** and solve the undamped($c=0$) spring(k)-mass(m) system with external force

$F = F_0 \cos \omega t$. Please discuss the following two cases: (a) $\omega \neq \omega_0$; (b) $\omega = \omega_0$ (this case is

known as the resonance). ($\omega_0 = \sqrt{\frac{k}{m}}$ is the natural frequency). Show the details of your work. (20 points)

3. Matrix $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

(a) Diagonalize \mathbf{A} . Show the details of your work. (6 points)

(b) Use the result in (a) to find \mathbf{A}^4 . (6 points)

4. Evaluate the following vector integrals.

(a) $\int_C \mathbf{F} \cdot d\mathbf{r}$, $\mathbf{F} = 4y \hat{\mathbf{i}} + 3x \hat{\mathbf{j}} + 2z \hat{\mathbf{k}}$ C : the intersection of $x^2 + y^2 + z^2 = 5$ and $z = 1$ clockwise. (8 points)

(b) $\iint_S \mathbf{F} \cdot \hat{\mathbf{n}} dA$, $\mathbf{F} = y \hat{\mathbf{i}} + x \hat{\mathbf{j}} + z \hat{\mathbf{k}}$ S : $x^2 + y^2 = 1$, $-1 \leq z \leq 1$. (8 points)

九十二學年度 原子科學 系(所) 甲 組碩士班研究生招生考試

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5. Find the temperature $u(x,t)$ in a bar of length L . This bar is perfectly insulated, also at the ends. The initial temperature distribution is $f(x)$. What is the temperature distribution as $t \rightarrow \infty$? (16 points) Given: Heat equation $\frac{\partial u(x,t)}{\partial t} = c^2 \nabla^2 u(x,t)$
6. Find all Laurent series of $\frac{4z-6}{z^3-4z^2+3z}$ with center 0, please determine the precise region of convergence for each series. (6 points)
7. Evaluate the following integrals by using the residue theorem.

(a) $\oint_C \frac{4z-6}{z^3-4z^2+3z} dz$, and $C: \left| z - \frac{1}{2} \right| = 1$ clockwise. (6 points)

(b) $\int_0^\pi \frac{d\theta}{25-24\cos\theta}$ (6 points)