九十一學年度 原子科學 系（所） 丙 組碩士班研究生招生考試

科目 應用數學 科號 3402 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. Solve the following initial value problems.

   (a) $[(x+1)e^x - e^y]dx = x\,e^y dy$, $y(1) = 0$ (6%)

   (b) $x^2y'' - 4xy' + 4y = 40x^{-4}$, $y(1) = 5, y'(1) = 0$ (6%)

   (c) $y'' + 9y = r(t)$, $y(0) = 1, y'(0) = 4$ (10%)

$$r(t) = \begin{cases} 8\sin t & 0 < t < \dfrac{\pi}{2} \\ 0 & t > \dfrac{\pi}{2} \end{cases}$$

2. Experiments show that a radioactive substance decomposes (this phenomenon is called decay) at a rate proportional to the amount present. Please set up a mathematical model (a differential equation) of the physical process, and solve it (assuming initial amount is N_0, and the decay constant is $\lambda>0$). The half-life of a radioactive substance is defined as the time in which half of a given amount will disappear. What is the half-life of this radioactive substance? Please show the details of your work. (10%)

3. For a 3×3 matrixt $\mathbf{A} = \begin{bmatrix} 4 & -6 & -6 \\ 0 & -2 & 0 \\ 1 & -1 & -1 \end{bmatrix}$.

   (a) Find the eigenvalues and eigenvectors of **A**. (6%)

   (b) Diagonalize **A**. Show the details of your calculation. (6%)

   (c) Find $\mathbf{A}^{-1}$. (6%)

4. A ball rotates clockwise around the x-axis with constant angular velocity ω. Its initial position is (5,4,3) of the right-handed Cartesian coordinates. Please find the position vector $\mathbf{r}(t)$ of the ball center, and the corresponding velocity and acceleration. (10%)

5. Find the eigenvalues and eigenfunctions of the following problem.

$y'' + \lambda y = 0$, B.C. $y(-\pi) = 0$, $y(\pi) = 0$.

Verify the orthogonality of the eigenfunctions by direct calculation. (15%)

6. Find the Fourier cosine and sine series of the given function.

$$f(x) = \begin{cases} \dfrac{2kx}{L} & 0 < x < \dfrac{L}{2} \\ \dfrac{2k}{L}(L - x) & \dfrac{L}{2} < x < L \end{cases}$$

Use the result to evaluate $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$ (15%)

7. Use the residue theorem to evaluate the integral.

$\oint_c \frac{2z+3}{z^2 - z} dz$, C: $|z| = 2$ clockwise. (10%)