九十學年度 原子科學 系（所） 甲/丙 組碩士班研究生招生考試

科目 應用數學 科號 3302 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

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(12%) 1. Consider a first order linear system $\mathbf{y}' = \mathbf{A}\mathbf{y} + \mathbf{g}$,

where $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, and $\mathbf{g} = \begin{bmatrix} 6e^{2t} \\ 3e^{2t} \end{bmatrix}$.

(6%) (a) Find the eigenvalues and eigenvectors of **A**. Use the basis of eigenvectors to diagonalize **A**.

(6%) (b) Find the general solution.

(18%) 2. Solve the following initial value problems.

(8%) (b) $y'' - y' - 2y = 3e^{-t}$

I.C. $y(0) = 2$; $y'(0) = 0$

(10%)(a) $y'' + y' - 2y = r(t)$

where $r(t) = \begin{cases} 3\sin t - \cos t & 0 < t < 2\pi \\ 3\sin 2t - \cos 2t & t > 2\pi \end{cases}$

I.C. $y(0) = 1$; $y'(0) = 0$

(16%) 3. Assume an object is moving along the curve C: $x^2 + y^2 = 1, z = 2y$.

(6%) (a) Use position vector $\mathbf{r}(t)$ to represent its motion. Find the velocity and acceleration of this object.

(4%) (b) Find the points where the object has maximum speed. Find the corresponding tangential and normal acceleration at those points.

(6%) (c) If we apply a force $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ to this object. Find the work done by **F** in the displacement along the curve C.

(10%) 4. Use the Gauss's divergence theorem to evaluate the volume of a sphere with radius a. Check your answer with the value $\frac{4}{3}\pi a^3$.

(12%) 5. Find the eigenvalues and eigenfunctions of the following problem:

$$y'' + \lambda y = 0, \qquad B.C.\ y(0) = 0,\ y(L) = 0.$$

Verify the orthogonality of the eigenfunctions.

(14%) 6. Use the result in problem 5 to solve the one-dimensional heat equation.

$$\frac{\partial u(x,t)}{\partial t} = c^2 \frac{\partial^2 u(x,t)}{\partial x^2}, \quad B.C.\ u(0,t)=0,\ u(L,t)=0; \quad I.C.\ u(x,0)=f(x).$$

(18%) 7. Evaluate the following integrals:

(6%) (a) $\int_0^\infty \frac{dx}{1+x^4}$

(6%) (b) $\int_0^\infty e^{-x^2}\, dx$

(6%) (c) $\oint_C \frac{z-23}{z^2-4z-5}\, dz$ C: $|z-1|=3$ counterclockwise