1. Please solve the following initial value problems.

(A) 5%

$$\frac{dy}{dx} + 2xy = 4x, \qquad y(0) = 3$$

(B) 5%

$$2x\frac{dy}{dx} - 10x^3y^5 = y, \qquad y(1) = 1$$

2. Please solve the following initial value problems.

(A) 5%

$$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = 0, \qquad y(1) = 1, \quad y'(1) = 1$$

(B) 5%

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = \frac{24}{x^2}, \qquad y(1) = 4, \quad y'(1) = -1$$

3. Please find a general solution for the following differential equations.

(A) 5%

$$\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} = e^x$$

(B) 5%

$$x^3\frac{d^3y}{dx^3} + x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^{-2}$$

4. Please solve the following initial value problem. 10%

$$\frac{dy_1}{dx} = -y_1 + 4y_2 + 6e^x - x + 11$$

$$\frac{dy_2}{dx} = 3y - 2y_2 - 6e^x + 3x - 6 \qquad y_1(0) = 4, \; y_2(0) = 0$$

5. Please find f(x) if its Laplace transform is equal to the following.

(A) 5%

$$\frac{s^2 - 6s + 4}{s^3 - 3s^2 + 2s}$$

(B) 5%

$$\frac{6s^2 - 26s + 26}{s^3 - 6s^2 + 11s - 6}$$

6. Given a matrix equation $\mathbf{Ax} = \mathbf{b}$ where **A** is a mxn coefficient matrix, **x** is a vector of n dimension, and **b** is a vector of m dimension. Please explain under what condition(s), the matrix system is called (1) overdetermined, (2) determined, and (3) underdetermined. 5%

7. Please use Gauss-Jordan elimination to determine the inverse of the following matrix. 5%

$$\begin{bmatrix} 371 & -76 & -40 \\ 36 & -7 & -4 \\ -176 & 36 & 19 \end{bmatrix}$$

8. Give $f = \exp(x^2-y^2)\sin(2xy)$, please determine (1) the gradient of f (i.e., $\mathbf{F} = \nabla f$), (2) the divergent of **F**, and (3) the curl of **F**. 5%

9. Please state the Divergence theorem of Gauss. 5%

10. Please evaluate the following integral. 10%

$$\oint_C \frac{e^{-3\pi z}}{2z+i}\,dz, \qquad C: \text{the boundary of the triangle with vertices } -1, 1, \text{and } -i$$

11. Please evaluate the following real integral. 10%

$$\int_{-\infty}^{\infty} \frac{\cos(4x)}{x^4+5x^2+4}\,dx$$

12. Please use the Fourier integral representation to show that 10%

$$\int_0^{\infty} \frac{\cos(xw)+w\sin(xw)}{1+w^2}\,dw = \begin{cases} 0, & \text{if } x < 0 \\ \dfrac{\pi}{2}, & \text{if } x = 0 \\ \pi e^{-x}, & \text{if } x > 0 \end{cases}$$