

1. Please solve the following initial value problems.

(A) 5%

$$\frac{dy}{dx} + 2xy = 4x, \quad y(0) = 3$$

(B) 5%

$$2x \frac{dy}{dx} - 10x^3 y^5 = y, \quad y(1) = 1$$

2. Please solve the following initial value problems.

(A) 5%

$$x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + 2y = 0, \quad y(1) = 1, \quad y'(1) = 1$$

(B) 5%

$$x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = \frac{24}{x^2}, \quad y(1) = 4, \quad y'(1) = -1$$

3. Please find a general solution for the following differential equations.

(A) 5%

$$\frac{d^3 y}{dx^3} + 3 \frac{d^2 y}{dx^2} = e^x$$

(B) 5%

$$x^3 \frac{d^3 y}{dx^3} + x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = x^{-2}$$

4. Please solve the following initial value problem. 10%

$$\frac{dy_1}{dx} = -y_1 + 4y_2 + 6e^x - x + 11$$

$$\frac{dy_2}{dx} = 3y_1 - 2y_2 - 6e^x + 3x - 6 \quad y_1(0) = 4, \quad y_2(0) = 0$$

5. Please find $f(x)$ if its Laplace transform is equal to the following.

(A) 5%

$$\frac{s^2 - 6s + 4}{s^3 - 3s^2 + 2s}$$

(B) 5%

$$\frac{6s^2 - 26s + 26}{s^3 - 6s^2 + 11s - 6}$$

6. Given a matrix equation $Ax = b$ where A is a $m \times n$ coefficient matrix, x is a vector of n dimension, and b is a vector of m dimension. Please explain under what condition(s), the matrix system is called (1) overdetermined, (2) determined, and (3) underdetermined. 5%

7. Please use Gauss-Jordan elimination to determine the inverse of the following matrix. 5%

$$\begin{bmatrix} 371 & -76 & -40 \\ 36 & -7 & -4 \\ -176 & 36 & 19 \end{bmatrix}$$

8. Give $f = \exp(x^2 - y^2) \sin(2xy)$, please determine (1) the gradient of f (i.e., $F = \nabla f$), (2) the divergent of F , and (3) the curl of F . 5%

9. Please state the Divergence theorem of Gauss. 5%

10. Please evaluate the following integral. 10%

$$\oint_C \frac{e^{-3\pi z}}{2z + i} dz, \quad C: \text{the boundary of the triangle with vertices } -1, 1, \text{ and } -i$$

11. Please evaluate the following real integral. 10%

$$\int_{-\infty}^{\infty} \frac{\cos(4x)}{x^4 + 5x^2 + 4} dx$$

12. Please use the Fourier integral representation to show that 10%

$$\int_0^{\infty} \frac{\cos(xw) + w \sin(xw)}{1 + w^2} dw = 0, \text{ if } x < 0$$

$$\frac{\pi}{2}, \text{ if } x = 0$$

$$\pi e^{-x}, \text{ if } x > 0$$