八十八學年度 原子科學 系(所) 甲、丙 組碩士班研究生招生考試

科目 應用數學 科號 3802 4003 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

(16%) 1. Solve the following differential equations.

(8%) (a) $y'' - y = r(t)$

$$\text{where } r(t) = \begin{cases} 1 & 0 < t < a \\ 0 & t > a \end{cases}$$

$$I.C. \quad y(0) = 0; \quad y'(0) = 0$$

(8%) (b) $y'' + y = \sin t$

$$B.C. \quad y(0) = 1; \quad y(\frac{\pi}{2}) = 1$$

(15%) 2. Please solve the Bessel's differential equation:

$$x^2 y'' + x y' + (x^2 - \nu^2) y = 0,$$

where ν is not an integer.

(12%) 3. For the following linear system

$$\begin{aligned} 2x + 5y + 3z &= 1 \\ -x + 2y + z &= 2 \\ x + y + z &= 0 \end{aligned}$$

(4%) (a) Find the inverse of the coefficient matrix.

(4%) (b) Find the eigenvalues and eigenfunctions of the coefficient matrix.

(4%) (c) Find the solution.

(16%) 4. For $\vec{F} = x^2\,\hat{i} + y^3\,\hat{j} + z(1-2x-3y^2)\,\hat{k}$,

S: the surface of $x^2 + y^2 + z^2 \le 4,\ 0 \le z \le 2$

(8%) (a) Evaluate the surface integral $\iint_S \vec{F} \cdot \hat{n}\, dA$,

where $\hat{n}$ is the outer unit normal vector of S.

(8%) (b) Evaluate the surface area $S = \iint_S dA$.

(10%) 5. For $f(x) = \begin{cases} xe^{-x} & \text{if } x \ge 0 \\ 0 & x < 0 \end{cases}$

(5%) (a) Find the Fourier transform of $f(x)$.

(5%) (b) Find the Fourier transform of $f(x-a)$.

(16%) 6. The heat equation is $\dfrac{\partial u}{\partial t} = c^2 \nabla^2 u$.

(3%) (a) What is the Laplacian of a function u ($\nabla^2 u$) in spherical coordinate?

(3%) (b) What is the Laplacian of a function u in cylindrical coordinate?

(10%) (c) Find the temperature $u(x,t)$ in a insulated copper bar of length L with

B.C. $u(o,t)=0;\ u(L,t)=0$

I.C.. $u(x,0)=f(x)$

(15%) 7. Evaluate the following integrals:

(5%) (a) $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^3}$

(5%) (b) $\displaystyle\int_{-\infty}^{\infty} \frac{\sin x}{x}\, dx$

(5%) (c) $\displaystyle\int_C \frac{\sin z}{z}\, dz$ C: counterclockwise around the unit circle