八十七學年度 原子科學系 系（所） 甲 組碩士班研究生入學考試

科目 應用數學 科號 4102 共 3 頁第 1 頁 *請在試卷【答案卷】內作答

1. 12%

Consider the matrix

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(a) Find the eigenvalues and eigenvectors of A.
(b) Diagonalize A.

2. 12%

Find the solution of the following linear system.

$$y'_1 = -3y_1 - 4y_2 + 5e^t$$

$$y'_2 = 5y_1 + 6y_2 - 6e^t$$

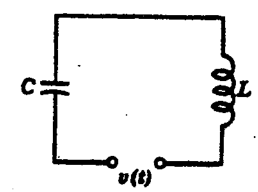
$$I.C. \quad y_1(0) = 1, \quad y_1(0) = 1$$

3. 14%

Let S be the closed surface consisting of the surface S_1 of the cone $z = \sqrt{x^2 + y^2}$ for $x^2 + y^2 \le 1$ and the flat cap S_2 consisting of the disk $x^2 + y^2 \le 1$ in the plane $z = 1$. The velocity vector is $\vec{F}(x,y,z) = x\vec{i} + y\vec{j} + z\vec{k}$. Verify Gauss's theorem by calculating the flux through S.

4. 12%

Find the current in the LC-circuit, assuming L=1 henry, C=1 farad, zero initial current and charge on the capacitor, and $v(t) = 1 - e^{-t}$ if $0 < t < \pi$ and 0 otherwise.



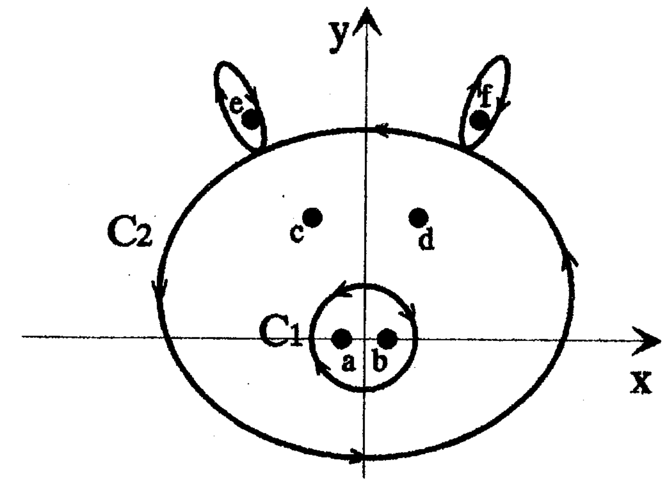
5. (10 points)

Write down the polar form of the complex number $(1+i)^{1-i}$, where $i \equiv \sqrt{-1}$ is the imaginary unit. Indicate its argument with the principal value.

6. (25 points)

Consider the following "pig head" in the complex plane shown below: C_1 is the unit circle C_1: $|z|=1$, *negatively* oriented, and C_2 is a contour enclosing C_1. The two "pig ears" enclosing points *e* and *f* belong to C_2. The coordinates of *a*, *b*, *c*, *d*, *e*, *f* are:

$a=-0.5+0i$, $b=0.5+0i$, $c=-1+2i$, $d=-\bar{c}=1+2i$, $e=-2+4i$, $f=-\bar{e}=2+4i$, where $i \equiv \sqrt{-1}$ is the imaginary unit.



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Evaluate the following line integrals and explain your answers

a. $\oint_{C_1} z dz$, (2 points)

b. $\oint_{C_1} \bar{z} dz$, (2 points)

c. $\oint_{C_1} \frac{1}{(z-a)^2(z-b)} dz$, (5 points)

d. $\oint_{C_1} \frac{1}{(z-a)^2(z-c)} dz$ (5 points)

e. $\int_{C_1+C_2} \frac{1}{(z-a)^2(z-b)} dz$, (5 points)

f. $\int_{C_1+C_2} \left(\frac{324}{z-a} + \frac{-475}{z-b} + \frac{2}{z-c} + \frac{3}{z-d} + \frac{1}{z-e} + \frac{-3}{z-f}\right) dz$. (6 points)

7. (15 points)

Static electric potential in two-dimensional free space satisfies the Laplace equation

$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$. Find the potential $V(x,y)$ in the box region $0 \le x \le a, 0 \le y \le b$, subject to the boundary conditions $V(x=0,a;y)=0$, $V(x;y=0)=0$, and $V(x;y=b) = V_0 \sin\left(\frac{7\pi}{a}x\right)$.