Problem 1 (15%)

Find the energy $U = (C\,V^2)/2$ of the parallel plate by using $U = \dfrac{\varepsilon_o}{2}\int (\vec{E}\cdot\vec{E})dv$.

Problem 2. (20%)

For the following cases, please draw a discrete charge distributions and write down the r dependent of the potential of each case

i)   Pure monopole

ii) Pure dipole

iii) Pure quadrupole

iv) Pure octopole

Problem 3. (20%)

In the discussion of the dipole approximation for an arbitrary charge distribution, we usually keep only the first order terms.   Please write down two second order terms.   You should show me where they come from.

Problem 4 (20%)

A charge q in front of an infinite metal plane with a distance d,

i)   Please find the induced surface charge density σ as function of position on the metal.(15%)

ii) Please find the force between the metal plane and the charge. (5%)

Problem 5 (25%)

For an uniform magnetic field $\vec{B} = B_o\,\hat{z}$

i)   Please find two of its vector potential $\vec{A}$ satisfied the condition of $\nabla\cdot\vec{A} = 0$. (10%)

ii) One of the vector potential could be $\vec{A} = [\,ax,\ 0,\ B_o y]$, with a an arbitrary constant However, its divergence is not zero. Please find a function Ψ, such that $\vec{A'} = \vec{A} + \nabla\,\Psi$ and $\nabla\cdot\vec{A'} = 0$. (15%)