

八十六學年度 原子科學 系(所) 甲 組碩士班研究生入學考試

科目 近代物理 科號 4203 共 二 頁第 一 頁 \*請在試卷【答案卷】內作答

1. 20%

Consider a rod lying along the  $X$  axis and at rest in reference frame  $S$ . The position coordinates of its ends are 0 and  $x$ . Two clocks are placed at the two ends of the rod. There is another reference frame  $S'$  which moves along  $XX'$  direction with a velocity  $0.6c$  with respect to the rod.

- Determine the length of the rod when viewed from  $S'$ . (5%)
- How to make the two clocks synchronized? (5%)
- The two clocks emit simultaneously a flash of light in the  $Y$  direction. These two flashes are received in  $S'$  by two of the counters spaced along the  $X'$  axis. How far apart are these two counters? (5%)
- Please justify the sameness or difference of the answers from (a) and (c). (5%)

2. 20%

The wavefunction of a particle in an infinite square well potential of width  $2a$  is:

$$\varphi = A\left(5\cos\frac{\pi x}{2a} + 3\cos\frac{3\pi x}{2a} + 4\sin\frac{2\pi x}{a}\right)$$

and  $\varphi = 0$  outside the well.

- Find  $A$ . (5%)
- What are the possible values you would obtain in a measurement of the total energy? What is the probability of each of them? (10%)
- What is the average total energy? (5%)

3. 10%

Prior to the discovery of the neutron there was Proton-Electron Hypothesis: a nucleus of mass number  $A$  and atomic number  $Z$  contained  $A$  protons and  $A-Z$  electrons. Please use the concept of uncertainty principle to judge this hypothesis.

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4. 10%

Compare the spectrum of a hydrogen atom to that of a deuterium atom.

$$m_p = 9.109 \times 10^{-31} \text{ kg}, \quad m_n = 1.672 \times 10^{-27} \text{ kg}, \quad m_e = 9.109 \times 10^{-31} \text{ kg}$$

$$R_H = 109737 \text{ cm}^{-1}$$

5. 25%

Some eigenfunctions for the hydrogen atom are

$$\psi_{100} = Ae^{-r/a_0}$$

$$\psi_{211} = B \frac{r}{a_0} e^{-r/2a_0} \sin \theta e^{i\phi}$$

$$\psi_{221} = C \frac{r^2}{a_0^2} e^{-r/2a_0} \sin \theta \cos \theta e^{i\phi}$$

where  $A$ ,  $B$ , and  $C$  are constants.

- Write out the eigenfunctions after parity operation. (6%)
- Use parity consideration to identify allowed electric dipole transitions among these eigenfunctions (5%)
- What are the expectation values of  $L_x$ ,  $L_y$ ,  $L_z$ , and  $L^2$  of  $\psi_{211}$ ? (8%)
- What is the magnitude of the angular momentum and its  $z$  component if the electron is in the state described by  $\psi_{211}$ ? (6%)

6. 15%

Consider a one-electron atom with energy  $-1.51Z^2$  eV, where  $Z$  is the atomic number.

- What is the number of degeneracy of quantum states corresponding to that energy? (5%)
- An external magnetic field, weaker than the internal field, is applied. Write down all the possible values of the good quantum numbers. (5%)
- Do the same as in (b) for the case a very strong external magnetic field is applied. (5%)