八十六學年度 原子科學 系（所） 甲 組碩士班研究生入學考試

科目 應用數學 科號 4202 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

(10%) 1. Solve the initial value problem.

$$y'' + 4y' + 3y = r(t), \quad I.C. \quad y(0) = 0, \; y'(0) = 2.5$$

$$\text{where} \quad r(t) = \begin{cases} 2e^{-t} & \text{if } 0 < t < 1 \\ 0 & \text{if } t > 1 \end{cases}$$

(10%) 2. Find the solution of the following linear system.

$$y_1' = -y_2 + \cos t - \sin t$$

$$y_2' = -y_1 + \cos t + \sin t$$

$$I.C. \quad y_1(0) = -1, \quad y_2(0) = -6$$

(10%) 3. Find the eigenvalues and eigenfunctions.

$$(e^{2x} y')' + e^{2x}(\lambda + 1)y = 0$$

$$B.C. \quad y(0) = 0, \quad y(\pi) = 0$$

Do these eigenfunctions form an orthogonal set?

(14%) 4. $\text{Matrix } \mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(3%) (a) Find $\mathbf{A}^T$. What type of matrix is this (symmetric, Hermitian,....)?

(5%) (b) Find the eigenvalues and eigenvectors of **A**.

(3%) (c) Find a matrix **X** which can diagonalize **A** (*i.e.* $\mathbf{X}^{-1}\mathbf{AX}=\mathbf{D}$).

(3%) (d) Find the diagonal matrix **D** in (c).

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(10%) 5. For $\vec{F} = (x+y)\,\hat{i} + (y+z)\,\hat{j} + (z+x)\,\hat{k}$,

S: the surface of a cylinder $x^2 + y^2 = 4$, $2 \geq z \geq 0$

Evaluate the surface integral $\iint_S \vec{\mathbf{F}} \cdot \hat{n}\, dA$,

where $\hat{n}$ is the outer unit normal vector of S.

(10%) 6. Find the Fourier transform of $f(x)$

$$\text{where} \quad f(x) = \begin{cases} xe^{-x} & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases}$$

What is the Fourier transform of $f(x-a)$?

(16%) 7. Solve the two-dimensional wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right)$$

$B.C.$ $\quad u(R,t) = 0$

$I.C.$ $\quad u(r,0) = f(r); \quad \left.\frac{\partial u}{\partial t}\right|_{t=0} = g(r)$

(20%) 8. Evaluate the following integrals:

(10%) (a) $\int_C \frac{z^2 - 1}{z^2 + 1}\, dz$ where C: the circle of $|z - i| = 1$ (clockwise)

(10%) (b) $\int_{-\infty}^{\infty} \frac{dx}{1 + 4x^2}$