國 立 清 華 大 學 命 題 紙

八十五學年度 原子科學 系（所） 甲 組碩士班研究生入學考試

科目 應用數學 科號 4002 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

(16%) 1. Solve the following initial value problems.

(8%) (a) $y'' - 5y' + 6y = r(t)$, $y(0)=1$, $y'(0)=-2$

where $r(t) = \begin{cases} 4e^t & 0 < t < 2 \\ 0 & t > 2 \end{cases}$

(8%) (b) $\begin{cases} y_1' = 6y_1 + 9y_2 \\ y_2' = y_1 + 6y_2 \end{cases}$

$i.c.$ $y_1(0) = -3$, $y_2(0) = -3$

(15%) 2. Legendre polynomial $P_n(x)$ satisfies the Legendre's differential equation:

$$(1 - x^2)\, y'' - 2x\, y' + n(n+1)\, y = 0.$$

Please solve for $P_n(x)$, and show that $P_n(x)$ forms an orthogonal set on [-1,1].

(14%) 3.

Matrix $\mathbf{A} = \begin{bmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{bmatrix}$

(6%) (a) Find the eigenvalues and eigenvectors of **A**.

(4%) (b) Find the trace of **A**.

(4%) (c) Find the inverse of **A**.

(10%) 4. For $\bar{F} = (x+z)\,\hat{i} + (y+z)\,\hat{j} + (x+y)\,\hat{k}$,

S: $x^2 + y^2 + z^2 = 1$, $z \geq 0$

Evaluate the surface integral $\iint_S \bar{F} \cdot \hat{n}\, dA$,

where $\hat{n}$ is the outer unit normal vector of S.

八十五學年度 原子科學 系（所） 甲 組碩士班研究生入學考試
科目 應用數學 科號 4002 共 2 頁第 2 頁 *請在試卷【答案卷】內作答

(15%) 5. Solve the one-dimensional heat equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

boundary condition $u(0,t) = 0,\ u(L,t) = 0$ for all t,

initial condition $u(x,0) = f(x)$.

(20%) 6. Evaluate the following integrals:

(12%) (a) $\int_C f(z)dz$, where

$$f(z) = \frac{1}{z-1},\quad \text{C: the circle } |z-1| = 1 \text{ (clockwise)}$$

Please use both direct line integral method and Cauchy Integral formulate to evaluate.

(8%) (b) $\displaystyle\int_0^{2\pi} \frac{\sin\theta}{2+\cos\theta}\, d\theta$

(10%) 7. X is the random variable which has the density

$$f(x) = \begin{cases} x/2 & \text{if } 0 \le x \le 2 \\ 0 & \text{otherwise.} \end{cases}$$

and the random variable $Y = 2X + 1$.

(3%) (a) Find the mean of Y.

(3%) (b) Find the variance of Y.

(4%) (c) Find the probability of finding Y in the interval $2 < Y \le 3$, $P(2 < Y \le 3)$.